Event-triggered control for linear systems with performance and rate guarantees

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Published: 20/06/2017

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An Approximate Dynamic Programming Approach

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NWO | Applied and Engineering Sciences

This work is supported by the Innovational Research Incentives Scheme under the VICI grant "Wireless control systems: A new frontier in automation" (no. 11382) awarded by NWO (Netherlands Organization for Scientific Research) Applied and Engineering Sciences (former Dutch Science Foundation, STW).

A catalogue record is available from the Eindhoven University of Technology Library.
ISBN: 978-90-386-4293-2

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PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de
Technische Universiteit Eindhoven, op gezag van de
rector magnificus, prof.dr.ir. F.P.T. Baaijens, voor een
commissie aangewezen door het College voor
Promoties, in het openbaar te verdedigen
op dinsdag 20 juni 2017 om 16.00 uur

door

Behnam Asadi Khashooei

geboren te Teheran, Iran

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Het onderzoek dat in dit proefschrift wordt beschreven is uitgevoerd in overeenstemming met de TU/e Gedragscode Wetenschapsbeoefening.

Contents

Preface

A giant leap for a man, a small step for mankind!

Four years has passed and as I write these words, I tend to dive into the memories of my experiences within this somehow intense period, leaving the home sweet home for a wonderland to study and work in an amazingly dynamic environment and to grow. During this period, I learned a lot and unlearned even more for which I am truly grateful. I would like to thank many people for their presence and support during this period without which I wasn't able to achieve what I strove for.

Maurice, I've been always touched by your warm presence and openness in our meetings and am grateful for all your supportive, considerate and genuine advises as well as your commitment to high scientific standards, which were invaluable sources of inspiration for me during these years.

Duarte thank you for guiding me to grow as a scientist, for introducing me to the beauty of dynamic programming and stochastic control, and for your unconditional support, kindness, and patience which shined through our interactions.

Nick thanks for being there in the challenging phase of settling down in a new environment with all your warm and inspiring presence. Menno thanks for sharing with me your interesting ideas on science, consciousness, and life in general and receiving me in your warm home with lovely Ellen. Bas, bro, no words can explain my gratitude to you being there in difficult, happy and crazy times and encouraging me to keep going on in difficult times. Masoud, I learned a lot from your patience, wisdom, and fair judgments and always enjoy your company, thank you Refigh. Victor thank you for sharing with me some of your enthusiasm for the classical music and science and for being a true friend. Xi thank you for all the adventures you accompanied me from watching the sunrise in Linz, snowboarding, city trips, and sharing with me your artistic, cultural viewpoints and being a great listener. Robert the cyclist! thank you for listening to all my theories and for introducing me to the world of cycling.

Emanuel, thank you for being a great officemate and friend, I won't forget your skills in drone piloting and unreal tournament! Niek, we started our Ph.D. together and shared a lot during this period from the unforgettable trip to Lucca to the awesome thanksgiving party, thank you for being part of these moments. Tom thank you for all the lunch-time gyms, the fascinating one-hour work trip by motorbike, and the unforgettable morning run! Sava thank you for the coffee corner talks, the movie nights and restaurant gatherings. Mathijs thank you for your company for dinners at the auditorium, your kind and thoughtful advises and your sense of humor! Nikos, I always enjoyed the deep talks we had, thank you man! Isaac, Alejandro, Mahmoud, Cesar, Haitao, and Bayu, my squash buddies, it's been always a pleasure to play and share thoughts and experiences with you. Carlos, Alper, my old officemates! thank you for your kind help during the first year of my Ph.D. Mahnaz and Nilgoun, thank you for all the lunch-time laughs and discussions that reminded me of Iran. Bas and Nitish, the last part of this dissertation wasn't possible without your collaborations, thank you for your assistance and hard work. I would also like to thank all the people in CST and D&C groups, in particular, Frank, Joost, Eelco, Stephan, Elise, Lennart, Rolf, Robbert, Rob, Michiel, Emilia, Jurgen, Rishi, Suraj, Hadi, Aida, Geertje, and Petra, with whom I greatly enjoyed talking, traveling and dining out during these years.

I extend my sincere gratitude to the members of my reading committee. Henk Blom, Siep Weiland, Daniel Quevedo, and Tamas Keviczky thanks for reviewing this dissertation and for taking part in my promotion.

Finally, I would love to thank my family, friends, and teachers in Iran who always have been the sources of encouragement and inspiration for me. Especially, I'd like to thank three persons that words can not describe how much I am grateful to have them in my life: Shohreh, my teacher of compassion, patience, and simplicity in life, Mehrnoosh, my genuine friend and critic, and Anneke, my schat! Thank you!

Behnam Asadi
Spring, 2017

Chapter 1

Introduction

Introducing a work is always an art, the art of storytelling, starting with a vague and general idea and uncovering, word by word, the specific details of the story. In this chapter, we begin with a general overview of the challenges in the new era of communication and control, and then we dive into a particular body of work pertaining to the design of dynamic communication protocols for control systems which is commonly referred to as event-triggered control (ETC).

1.1 Motivation

We are living in an information-rich era, where almost every aspect of our lives is influenced by ubiquitous information networks. Not long ago the world wide web was just a new medium amongst others to ease connectivity, but now it is a large global information grid to which nearly half (45.6% by 2015) of the world's population is connected [1].

Whether believing the opportunistic view of Metcalfe's law [2] stating that the value of each network grows quadratically by the number of its users (i.e. $O(n^2)$ with n users) or having a more realistic viewpoint that the growth in value is more of an order of $n\log(n)$ [3], the connectivity is financially very attractive. As a result, the internet has become not only the most dominant networking medium but also an essential aspect of daily life influencing our health, social behavior, educational systems, economy, security, industries and so on [4]. This also lead to the concept of the Internet of Things (IoT) defined in [5] as:

> *"In what's called the Internet of Things, sensors and actuators embedded in physical objects - from roadways to pacemakers - are linked through wired and wireless networks, often using the same Internet Protocol (IP) that connects the Internet. These networks churn out huge volumes of*

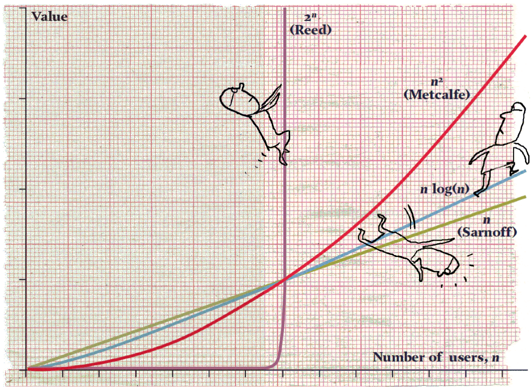


Figure 1.1. How much more valuable does a network become as it grows with respect to its users? Source: "Metcalfe's law is wrong -communications networks increase in value as they add members- but by how much?", B. Briscoe, A. Odlyzko and B. Tilly. © [2006] IEEE.

> *data that flow to computers for analysis. When objects can both sense the environment and communicate, they become tools for understanding complexity and responding to it swiftly. What's revolutionary in all this is that these physical information systems are now beginning to be deployed, and some of them even work largely without human intervention."*

This situation is unparalleled in human history and, as a consequence, the study of networks and their effects have become a scientific, and technological imperative for the 21st century [6]. This is also the case for control science, which is the field of engineering providing methods and principles to design systems that operate automatically in a dynamic environment in order to obtain a desired performance. In fact, the rapid growth in the usage of networked (wireless/wired) sensors and actuators creates many challenges and opportunities for control science. These challenges can be divided into two areas being the control of networks and the control over network [8], which are explained in more detail next.

Many control systems operate in real-time, that is, there is a tight time window to gather and process data, and update the control actions. A real-time application usually consists of several real-time tasks, which are activated either at regular intervals called the period (periodic tasks) or at the time of an event (aperiodic or event-driven tasks) [9]. A typical real-time control system consists of a plant, a digital controller and communication interfaces including sensors, samplers, holders, and actuators. The plant is often a continuous-time process whose behavior is monitored via sensors and is manipulated through actuators. Since the controller is digital and the communication network is packet-based, discrete-time samples of continuous-time measurement data are sent to the con-

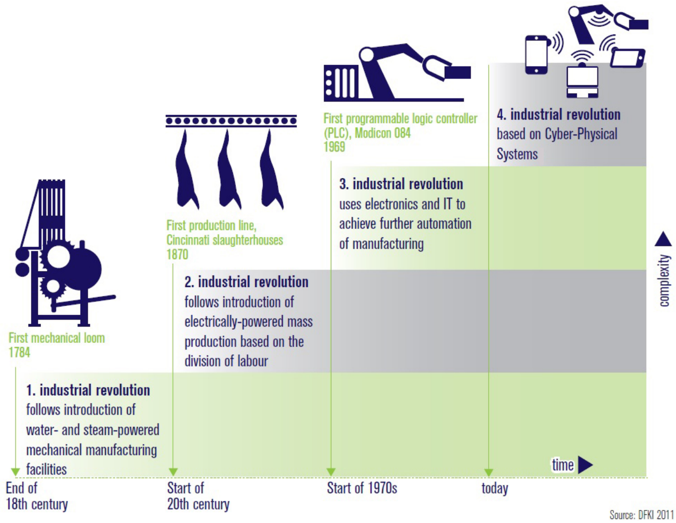


Figure 1.2. The four stages of the Industrial Revolution: The concept of IoT has found its way into many industries especially the manufacturing environment leading to *Industrie 4.0*, a fourth industrial revolution [7]. Source: the German Research Center for Artificial Intelligence. © [2011] DFKI GmbH.

troller. The controller processes this data to generate control actions, which in turn are sent to the holding systems, where the digital to analog conversion takes place and a continuous-time actuation signals are generated. Therefore, in these control systems, the closed-loop model contains both continuous-time and discrete-time signals, as such, these systems are known as sampled-data control systems [10]. Extensive research has been conducted on sampled-data systems especially in the 90's. However, in the context of sampled-data systems, all the sensors are considered to be sampled, and all the actuators are assumed to be updated at predefined sampling periods whether it is a global sampling period as in single-rate control or multiple sampling periods as in multi-rate control. This assumption typically does not hold in the context of control over networks. In fact, in the latter context, there are (i) limitations of communication resources, for example, when only a limited number of communication channels are available at each time instant, as well as (ii) network-induced problems such as packet dropouts, delays, etc., which also cause aperiodic and time-varying sampling and transmission processes. This asks for the development of new methods for analysis and design of control systems operating over communication networks. Control over networks deals with the challenges that arise in the control of these networked control systems (NCSs) [8, 11].

Control of networks, on the other hand, tackles the development of new techniques in protocol design, power management, routing, etc. This is a multidisciplinary field in which both the findings of the control and communication fields play an important role. Furthermore, next to the advancement in communication and computation technologies, there is a strong need for tools to manage the communication and computation resources in terms of bandwidth allocation and deadlines assignments, respectively. Also, as more and more devices, mostly battery-powered, are operating remotely, it has become crucial to design new tools to manage the energy resources.

In the sequel, we provide some examples of technologies and applications in which the control over/of network can be useful.

Examples of IoT's opportunities for control

Wireless sensor networks (WSNs)

A wireless sensor network consists of spatially distributed standalone devices that are connected through a wireless network. These devices have computation, communication, and sensing capabilities, which can be used to monitor physical phenomena, such as temperature, pressure, etc., in a large-scale system and to cooperatively transmit these data to a main monitoring unit [12].

Preferably these WSNs run with low-cost and highly scalable computation devices like Raspberry Pi [13, 14] that are battery-powered. Therefore, these computation devices have limited energy resources. On the other hand, wireless transmissions can be responsible for up to 98% of the overall energy consumption of the wireless devices [15]. Consequently, communication resource management can play an important role in increasing the lifetime of each sensor, leading to an improved overall performance of a WSN.

Cloud computing

An important application of resource allocation is in cloud computing, which can be seen as a byproduct of the Internet of Things. In cloud computing, a pool of scalable computation power, storage, and services are delivered on demand to clients over the Internet [16]. For example in the case of battery-powered devices, like smartphones, cloud computing can be very beneficial when the savings from offloading the computation exceeds the energy cost of the additional communication to the cloud center. This asks for smart resource allocation tools (see, e.g., [17,18]). Cloud computing also opens several opportunities for control, robotics and WSN applications. In fact, stand-alone, low-cost devices in the near future may send processing tasks to the cloud instead of computing these locally, enabling these devices to offload resource-intensive tasks such as image processing. However, cloud computing will also have a cost. Assuming that such a cost will be proportional to the required computational usage, it will be important to manage the access to these resources, similar to the communication resource management problems mentioned above.

Figure 1.3. Communicating cars, not long ago communication between cars was a dream. The use of 'tethering' permitted bulky recording equipment to follow in the vehicle behind without affecting the test vehicle. Source: Ford of Britain 100: Image of the week - 18/52 © [2011] Ford.co.uk

Communicating vehicles

Increasing road traffic has created many challenges in terms of road safety, air pollution, and energy consumption in many countries resulting in the advent and development of Intelligent Transportation Systems (ITS) technologies that contribute to improved traffic flow and safety. Not long ago exchanging sensor data between cars was only possible through tethering (Figure 1.3), but nowadays vehicles are empowered by advanced communication systems and computational resources enabling the communication between cars and, therefore, the implementation of ITS technologies. In fact, advanced wireless technologies enable Cooperative Adaptive Cruise Control (CACC) by extending the Adaptive Cruise Control (ACC) technology with the addition of information exchange between vehicles through Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) wireless communication [19, 20]. The CACC concept appears to improve traffic throughput by decreasing the inter-vehicle's distance. However, the full potential of CACC in increasing highways capacity is only visible when more than 60% of the vehicles apply CACC [19]. On the other hand, in dense traffic situations, intensive network usage degrade the reliability of the wireless network and increase the transmission delay which in turn can have a significant negative influence on the performance of CACC. Therefore, in the development of CACC, the allocation of communication resources will be extremely important [21].

1.2 Communication Resource Management

Efficient utilization of communication resources is one of the key challenges in many new networked control applications as also illustrated by the examples at the end of the previous section. To address this challenge, several solutions have

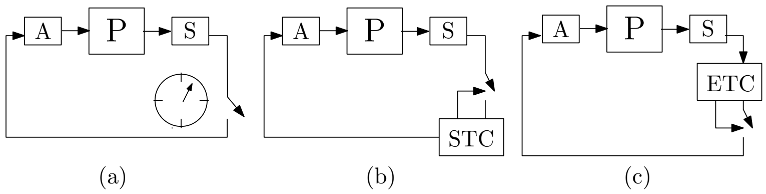


Figure 1.4. Illustration of different network-resource-utilization solutions in a single-loop control structure: (a) Periodic control in which the loop is closed at fixed periodic time instants, (b) self-triggered control in which the feedback-time instants are determined based on the plant model and the last transmitted data (e.g., sensor measurements), and (c) event-triggered control in which the control loop is closed whenever a certain criterion relying on current data is satisfied.

been introduced. We can roughly classify these solutions in two categories.

Open-loop/Static Solutions Periodic sampling and control is the most common approach. In this method, communication takes place at prespecified periodic time instants designed to meet the communication resource constraints as well as the performance of the control loop. It is clear that the periodic solutions are easy to implement. However, they lack the flexibility to efficiently utilize communication resources. For example, in the case of battery-powered sensors, this means that the sensors wake up every time instant of a periodic sequence and transmit sensor data even if the data has not changed when compared to the previous transmission instant. This solution typically requires unnecessary use of resources.

Closed-loop/Dynamic Solutions In closed-loop solutions for resource management, one needs to introduce an additional computational unit or decision maker that specifies, based on the available information, the time instants at which the transmission should happen. This decision maker is designed such that the transmissions only take place when needed to guarantee stability and performance properties, thereby reducing the utilization of the available communication resources. Roughly speaking, two different approaches for these methods have been proposed being a proactive approach known as self-triggered control (STC) and a reactive approach known as event-triggered control (ETC).

- *Self-triggered Control*
  In STC (see, e.g., [22–26]), each time instant at which the network is utilized, the next utilization time is also determined based on available information from sensors and the process model. This method for making transmission decisions is, roughly speaking, blind to the changes that may occur between the scheduled utilization times. Although in some safety-critical applications (e.g., vehicle platooning and CACC), this may

be a drawback, in many other applications with limited computation and energy resources it can provide significant benefits. For example in the case of battery-powered wireless sensors, STC requires that at each wakeup time of the sensor the next wakeup time is also computed. This allows the sensors to operate on standby power between these wakeup times thereby preserving valuable energy resources and extended the lifetime of the batteries.

- *Event-triggered Control*
  The fundamental idea behind ETC is that network utilization should be triggered by events inferred from the state or the output. This method is reactive as the available information, and in some cases the process model, are continuously used to determine if the resources should be utilized or not. The ETC can play an important role in communication resource management in safety-critical applications, where the system model is not accurate enough or where, due to the highly unpredictable occurrence of disturbances, it is necessary to monitor the process almost continuously. For example, in hazard detection systems based on wireless sensors [27], continuous monitoring of environment is necessary, but the transmissions of sensors' data are only required when a deviation from the desired behavior is observed or anticipated. Therefore, event-triggered algorithms can play a major role in managing energy and communication resources by transmitting data only when it is necessary.

The difference between these solutions is illustrated in Figure 1.4.

In this dissertation, the main focus is on the design of ETC algorithms using an optimization-based view-point. In the next section, we will give an overview of the current state of the art regarding ETC.

1.3 Event-triggered Control: A literature overview

Motivated by the necessity of addressing energy, computation, and communication constraints when designing feedback control loops, the pioneering works [28] and [29] portrayed some clear advantages of event-based control with respect to periodic control when handling these constraints. In subsequent works, systematic designs of event-based implementations of stabilizing feedback control laws were explored (see, e.g., [30–34]) and an emerging body of research work generalized these results leading to alternative approaches [35–42], including the case of output-feedback ETC, which has inherent complications, see, e.g. [43].

The works [28,30,32] proposed to trigger transmissions when the norm of the state or the error between the previously transmitted state and the current state exceeds a certain threshold (absolute triggering). In [28], it was shown that such a policy, also known as the absolute threshold policy, can outperform periodic control regarding the steady-state variance for the same average transmission rate for a scalar linear system. Another influential work [33], proposed to trigger transmissions when the norm of the error between the new and the previously transmitted data exceeds a weighted norm of the new state in order to guarantee a decrease rate for a Lyapunov function. The resulting policy is often referred to as relative threshold policy. In [44], mixed threshold policies that combine the relative and absolute threshold policies, were proposed. There, it was shown that the absolute threshold part, ensures a nonzero minimum inter-transmission time and thus guarantees a bound on the total number of transmissions. Recently, dynamic ETC in which the conditions to trigger transmissions take into account past state information was proposed in [45] which was further improved and extended to output-based triggering conditions in [37]. In another view point, the works [33, 36, 37, 43–45] can be classified as Lyapunov-based approaches (mostly for nonlinear/hybrid systems) in which the ETC policy is derived through a performance notion, categorized by $\mathcal{L}_p$ stability, Input-to-state stability (ISS), etc., while guaranteeing a lower bound for inter-event times.

On the other hand, ETC policies can also be obtained from optimal control formulations taking into account the closed-loop performance and the network usage (see, e.g., [34,38–40,46–51]). These formulations are more common for linear systems. The optimal design problem for single-loop control systems with an information-constrained feedback loop can be regarded as a two-person team decision problem. In fact, when an event-triggered decision-maker collocated with the sensors transmits data via the network to a remote controller collocated with the actuators, the controller and the event-triggered decision-maker take the role of individual decision-makers aiming at the optimization of a shared objective [52]. The closed-loop performance is typically defined in terms of a quadratic cost as in the celebrated LQR and LQG problems. As one of the earliest attempts in characterizing the optimal event-triggered policy, [49] formulates the optimal communication logic as a long term average cost optimization problem that can be solved using dynamic programming. In [53], it is shown that if the triggering policy is fixed in advance and does not use past control actions, then the certainty equivalent control is optimal for a quadratic cost function. In [52, 54], a detailed characterization of the optimal co-design of event-trigger and controller in the framework of linear quadratic control under three variants of resource constraints has been investigated. There it is shown that under a nested information structure, the set of policy pairs where the controller is a certainty equivalent controller is a dominating policy. Consequently, the co-design problem reduces to the joint design of an event-trigger policy and a state estimator. This joint optimization of the estimator and the event-trigger for scalar systems results in absolute threshold policies for event-triggered mechanism and a class of Kalman-like filters for the estimator [55]. In [56], it is shown that in the class of symmetric absolute threshold event-triggered policies, the optimal triggering mechanism can be obtained via dynamic programming. In [57], optimal triggering conditions under model uncertainty was investigated. There, by following a similar approach as in [52], a triggering mechanism via dynamic programming formulations has been proposed. In [58], utilizing a different performance index in terms of the second moment of a scalar stochastic linear system, an ETC mechanism is designed such that, in the presence of packet dropouts, the second moment of the state converges exponentially to a desired set in finite time. It is further shown that the proposed policy in [58] under mild conditions provides guaranteed bounds over the transmission rate.

Despite their analytical importance, the obtained dynamic programming formulations in the aforementioned works suffer from the curse of dimensionality. Therefore, most of the optimal triggering policies proposed in the literature are hard to implement in practice and lack the insight and the simplicity of the basic policies described in the pioneering works [28, 30, 32, 33].

Prompted by these facts in some works suboptimal event-triggered controllers with guarantees on the closed-loop performance and/or on the network usage have been proposed [26, 40, 46, 48, 53, 59, 60]. One can trace back this approach to the early work of [28], in which it was shown that the event-based sampling outperforms the periodic sampling regarding average variance of deviation error. In [59], a suboptimal absolute threshold policy is proposed. This policy incurs a performance within a factor of 6 of the optimal achievable performance. To obtain this performance, an algorithm to minimize an upper bound on the system performance using a quadratic approximate value function for the underlying Markov decision process has been provided. In [46], a suboptimal scheduling policy with a fixed decision horizon within which a specific number of transmissions are allowed has been proposed. There, the question is how to identify these transmission instants for upcoming horizon based on available information at the beginning of the horizon. The authors in [46] developed an approximate dynamic programming formulation and provided suboptimal solutions with guaranteed performance with respect to the periodic control at the same transmission rate. In [47], while the ETC policy is assumed to be given as a relative threshold policy, a suboptimal controller design procedure is carried out. This resulted in a guaranteed upper bound on the performance of the closed-loop system. In [48], a policy which decides between two sampling rates is introduced, and it is shown that the resulting performance is bounded. The fast sampling rate is selected when the state is outside of a desired predefined region to steer the system back to the desired region and a slow sampling which is active as long as the system is in the desired region. In [61], a robust event-triggered model predictive control (MPC) scheme based on Tube MPC methods was proposed. It was shown that the proposed ETC retain a guaranteed bound with respect to the periodically updated counterpart, with a reduced average amount of communication between the controller and the actuators.

In this dissertation, as we shall see, we will not aim at obtaining optimal

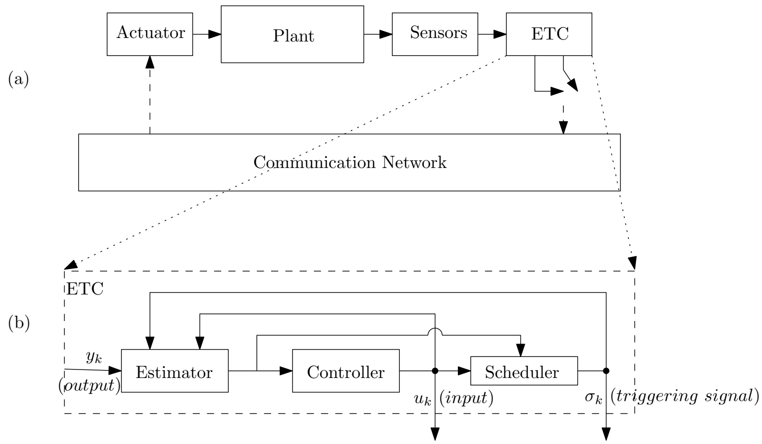


Figure 1.5. The schematic of the considered structure in Chapter 2: (a) The overall loop structure: The sensors are connected to the ETC block, which decides the transmission times and computes the control input at the transmission instant. (b) Inside the ETC: A scheduler decides based on the available sufficient statistics provided by the estimator and the event generator whether or not the computed control action will be transmitted.

solutions for the design of ETC policies, but rather suboptimal solutions that are simple to implement and intuitive with guaranteed performance. The research outputs that are presented in this dissertation are published in [62–65] and under review [66–69]. The content and contribution of theses publications are provided in the next section.

1.4 Research questions and dissertation contributions

We approached the suboptimal ETC design from two different perspectives. The first perspective was motivated by the following question:

How to design output-based ETC policies with performance guarantees?

In Part I of this dissertation, we aim to answer this question by proposing simple-to-implement policies with guaranteed performance bounds in terms of a quadratic cost.

In Chapter 2, we propose a new output-feedback stationary controller which is guaranteed to have a performance within a constant factor of the optimal periodic control performance (with all-time communication), while reducing the communication load. Performance is defined in terms of a quadratic average cost and we consider the co-design problem in which both the control inputs and the

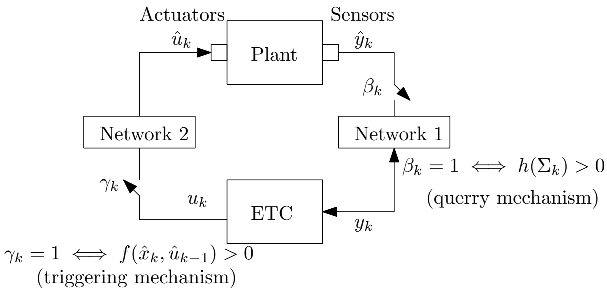


Figure 1.6. The overall setup and proposed policies in Chapter 3: Sensor query depends only on the Kalman filter covariance matrices Σ_k, while the control update transmissions are scheduled based on the Kalman filter state estimate $\hat{x}_k$, and the previously transmitted control input $\hat{u}_{k-1}$.

transmission decisions are designed simultaneously. The proposed transmission policy is based on a quadratic function of the Kalman filter state estimate, while the control input is determined by a linear function of this state estimate. The structure of the proposed ETC is shown in Figure 1.5. Furthermore, we discuss variants of the proposed policy. Interestingly, one of the proposed policies takes the form of a mixed threshold policy, as in [44], in which the absolute threshold term is proportional to the estimation error covariance and therefore to the magnitude of the disturbances. On the other hand, in the special case of no disturbances, it boils down to the relative threshold policy provided in [70], which in turn is connected to the policy provided in [33].

As an extension of Chapter 2, in Chapter 3, we consider an NCS in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The proposed ETC policies for sensor query and control input transmissions are derived using approximate dynamic programming (in particular, rollout techniques [71]) for an optimization problem which includes a quadratic performance cost defined in terms of state and input variables and penalty terms for transmissions. The new policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions. This is illustrated in Figure 1.6. In the online scheme determining the controller-to-actuator transmissions, the decisions are based on the state estimates, which are not known a priori and can be obtained via the time-varying Kalman filter. In the offline scheme for sensor query, transmissions are based on the covariances of the Kalman filter state estimates, which are known a priori. Interestingly, we can interpret this offline scheme as a policy in which transmissions occur when a function of the covariance of the Kalman filter exceeds a given threshold, which connects well to some policies proposed in this area of research (see, e.g., [72]). The main advantage of this approach is that we can show that this event-triggered policy

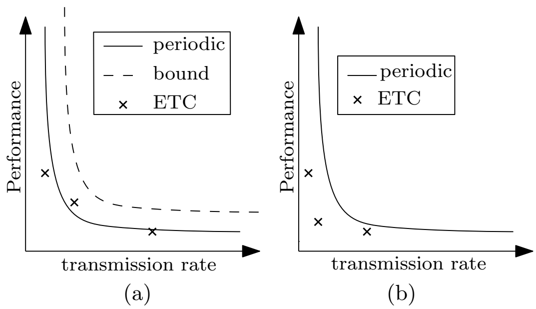


Figure 1.7. Comparison of research questions in terms of performance: (a) Design of ETC policies with guaranteed performance bounds in terms of the performance of the periodic time-triggered control (Part I of this dissertation) (b) Design of ETC policies that outperform the periodic time-triggered control (Part II of this dissertation).

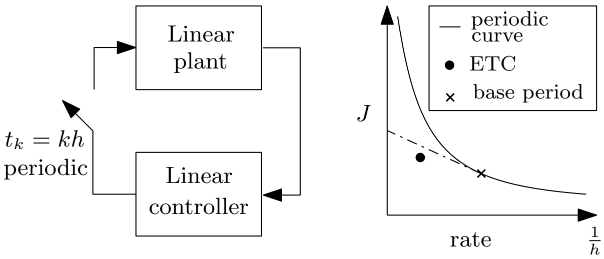
is stable (in a mean-square sense) and leads to performance guarantees in terms of the cost of all-time transmission policy, which was not the case in most other optimization-based ETC schemes. We discuss how this policy can be tuned to trade closed-loop performance guarantees and average transmission rates. Both schemes are simple to implement and have an insightful interpretation thereby increasing the acceptance of practitioners. In fact, as we will mention later, we validated the analytical results obtained in this chapter in an experimental setup presented in Chapter 6.

Although the focus on the first part was on simple and intuitive suboptimal event-triggered policies with performance guarantees, an interesting research problem that arose was:

> *How to develop policies that not only have guaranteed performance but also outperform the periodic time-triggered policies in terms of quadratic performance for the same average transmission rate?*

The difference between these two research questions is illustrated in Figure 1.7. To answer the second question, in Part II of this dissertation we propose ETC policies that are *consistent*. We say that an ETC policy is consistent if (a) the policy achieves a better closed-loop performance for the same average transmission rate or in other words, the same closed-loop performance at a lower average transmission rate, compared to the traditional periodic control and (b) generates no transmissions in the absence of disturbances.

In Chapter 4, we present a class of consistent policies for linear quadratic control, which takes the form of dynamic ETC recently proposed in the literature [45], [37]. In dynamic ETC, the conditions to trigger transmissions take into account past state information, in contrast with most policies where transmission decisions only depend on the present state information [39,46,48,49,54,59,73,74]. We consider a model for disturbances using Poisson jump processes, which, as



$$J = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}[\int_0^T (x'Qx + u'Ru)dt]$$

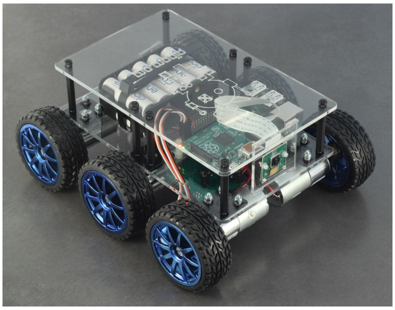
Figure 1.8. Illustration of the key property to establish consistency in Chapter 5. For periodic control, the trade-off curve between average quadratic performance vs. transmission rate is convex.

suggested in [75], can capture the more commonly used Wiener process model for linear quadratic Gaussian (LQG) control and are easier to handle mathematically. Moreover, these processes can also capture sporadic disturbance models. Our policy builds upon the trade-off curve between average inter-transmission time and average quadratic performance for periodic control, which we characterize explicitly. While it is not hard to find an example where the second property of consistency does not hold, the effectiveness of ETC policies make it nontrivial to find an example when ETC policies perform worse than periodic control. However, we manage to provide an example of a linear quadratic control problem for which a traditional ETC policy, where transmissions occur if the Euclidean norm of the error between the system's state and a state estimate exceeds a threshold, does not satisfy the first consistency property.

Motivated by this observation, in Chapter 5, we propose an absolute threshold ETC policy, using a weighted norm different from the Euclidean norm that is guaranteed to be consistent. The performance is measured by an average quadratic cost, as in the standard linear quadratic Gaussian (LQG) framework. While we consider continuous-time systems, the plant is only monitored periodically, at a fast rate, at which the transmission-triggering condition is checked. Therefore, this policy can be seen as a periodic ETC (PETC) policy a term introduced in [76]. The proposed solution builds upon a key result establishing the convexity of the trade-off curve between average cost performance vs. transmission rate for periodic control. In fact, we show that for the proposed ETC policy the pair (rate, performance) is below the tangent line of the periodic curve at the point corresponding to the base period of PETC policy. This property is illustrated in Figure 1.8.

In the last part of this dissertation, we focus on the experimental validation of some of the results of the previous setup to answer:

> *How to experimentally validate the developed results and deal with practical features such as packet dropouts in real scenarios?*


(a)

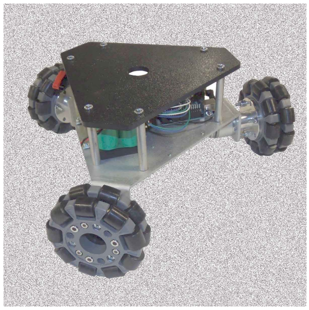

(b)

Figure 1.9. The ground robots used in the experimental validations in (a) Chapter 6 for 1D motion and (b) Chapter 7 for 2D motion.

In Chapter 6, we explore the validation of the proposed policy in Chapter 3 in the context of the remote control of a ground robot in 1D, see, Figure 1.9-(a). The experimental set-up consists of a robot, a wired camera, a control unit, and a wireless network as shown in Figure 1.6. The camera is used to obtain images of the robot in its workspace. The camera takes an image at designated time instants, which are defined by the controller, and sends the images to the controller. The controller receives the images from the camera and processes the images to get the estimated position of the robot with respect to a certain reference frame. Based on the estimated position of the robot the controller can compute new control actions. These computed control actions are sent through a wireless network to the actuators of the robot at designated time instants, which are defined by the controller, and the robot is steered towards the origin of the reference frame. The experiments validate the usefulness of the ETC policy derived in Chapter 3. In fact, the communication is reduced by 80% and 90% for the sensors' and the actuators' networks, respectively, while guaranteeing the performance bounds on the cost with respect to the all-time transmission policy.

As a follow up to the experimental validation presented Chapter 6, in Chapter 7, we considered an experimental scenario with the goal of having a robot track a predefined reference in 2D while preserving transmissions between the controller and the robot, see Figure 1.9-(b). In this scenario, we assumed that both actuators' and sensors' networks are subject to packet dropouts with known dropout probabilities. Prior to conducting the experiments, we developed two ETC policies corresponding to two different information structures. The proposed ETC policies depend on underlying characteristic of dropout model and obtain a closed-loop performance bound in terms of the performance of all-time transmission policy. Furthermore, the experiments validate the usefulness of the developed ETC policy and showed that the triggering can be reduced by 88% in the actuator's network, while achieving a performance withing the guaranteed bound.

1.5 Outline of the dissertation

This dissertation is divided into three main parts. Each part consists of two chapters each of which is based on a research paper and is self-contained with respect to other chapters and can be read independently.

Part I: Suboptimal ETC with guaranteed performance

Chapter 2: In this chapter, we propose an output-based ETC solution for linear discrete-time systems with a performance guarantee relative to periodic time-triggered control, while reducing the communication load. The performance is expressed as an average quadratic cost and the plant is disturbed by Gaussian process and measurement noises. We establish several connections with previous works in the literature discussing, in particular, the relation to absolute and relative threshold policies.

Chapter 3: In this chapter, we consider an NCS in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The goal is to reduce transmissions compared to all-time transmission policy while guaranteeing that the closed-loop performance is within acceptable bounds. Our approach extends a recent line of research where explicit ETC policies with performance guarantees are derived using approximate dynamic programming. The proposed policy in this new setting can be separated into an offline scheme for sensor query in which the transmission instants are computed a priori and an online scheme to schedule control input transmissions.

Part II: Consistent suboptimal ETC

Chapter 4: In this chapter, we propose a dynamic ETC strategy that is consistent for any linear system when performance is measured by an average quadratic cost. We say that an ETC policy is consistent if it achieves a better closed-loop performance than the traditional periodic control for the same average transmission rate and generates no transmissions in the absence of disturbances. A numerical example shows that these conditions may not be necessarily satisfied by an event-triggered policy for which transmissions are triggered if the Euclidean norm of the error between the system's state and a state prediction exceeds a given threshold.

Chapter 5: In this chapter, we propose an absolute threshold ETC policy in which a weighted norm (instead of Euclidean norm) of the error is used in triggering mechanism. We show that this ETC policy can always ensure consistency based an average quadratic performance measure. The proposed policy is of the class of periodic ETC policies that guarantee a fixed minimum inter-event time between transmission instants.

Part III: Experimental validation and application to ground robotics

Chapter 6: In this chapter, we extend and experimentally validate an ETC strategy presented in Chapter 3 for the remote point stabilization problem of a ground robot. This strategy specifies when transmissions should occur in both sensor-controller and controller-actuator channels, and guarantees a bound on performance measured by a finite-horizon quadratic cost. The experimental results are coherent with the simulation results and reveal that the proposed ETC leads to a tremendous data transmission reduction (up to 90%) with respect to periodic time-triggered control, with a minor performance loss.

Chapter 7: In this chapter, we propose an ETC policy with guaranteed performance under unreliable actuators' and sensors' links. The proposed policy is an absolute threshold policies whose weightings are influenced by the underlying characteristic of the packet dropout in the communication networks. An experimental setup for validation of the proposed algorithm has been developed. The setup consists of a ground omni-directional robot remotely controlled to follow a predefined trajectory. The experimental results show that using the proposed ETC the triggering can be reduced by 88% in the actuator's network, while achieving a performance withing the guaranteed bound.

Chapter 8: In this chapter, we conclude this dissertation and provide some interesting future research directions for the suboptimal design approach in ETC context.

1.6 Publications

The results presented in this thesis are based on the following publications.

- B. Asadi Khashooei, D.J. Antunes, and W.P.M.H. Heemels, "Rollout strategies for output-based event-triggered control," in *European Control Conference, 2015 European*, July 2015, pp. 2168-2173.

- B. Asadi Khashooei, D.J. Antunes, and W.P.M.H. Heemels, "Ouput-based event-triggered control with performance guarantees," *Automatic Control, IEEE Transactions on*, accepted for publication, Feb 2018.

- B. Asadi Khashooei, D.J. Antunes, and W.P.M.H. Heemels, "An event-triggered policy for remote sensing and control with performance guarantees," in *Decision and Control (CDC), 2015 IEEE 54th Annual Conference on*, Dec 2015, pp. 4830-4835.

- B. van Eekelen, N. Rao, B. Asadi Khashooei, D.J. Antunes, and W.P.M.H. Heemels, "Experimental validation of an event-triggered policy for remote sensing and control with performance guarantees," in *Event-based Control, Communication, and Signal Processing (EBCCSP), 2016 Second International Conference on.* IEEE, 2016, pp. 1-8.

- D.J. Antunes and B. Asadi Khashooei, "Consistent event-triggered methods for linear quadratic control," in *Decision and Control (CDC), 2016 IEEE 55th Annual Conference on*, Dec 2016, pp. 1358-1363.

- D. J. Antunes and B. Asadi Khashooei, "A consistent dynamic event-triggered policy for linear quadratic control," *submitted*, 2016.

- B. Asadi Khashooei, D.J. Antunes, and W.P.M.H. Heemels, "A consistent threshold policy for periodic event-triggered control," *submitted*, 2017.

- B. Asadi Khashooei, B. van Eekelen, D.J. Antunes, and W.P.M.H. Heemels, "Suboptimal event-triggered control over unreliable communication links with experimental validation,*Event-based Control, Communication, and Signal Processing (EBCCSP), 2017 Third International Conference on.* IEEE, 2017.

Part I

Suboptimal event-triggered control with guaranteed performance

Chapter 2

Output-based Event-triggered Control with Performance Guarantees

In this chapter[1], we propose an output-based event-triggered control solution for linear discrete-time systems with a performance guarantee relative to periodic time-triggered control, while reducing the communication load. The performance is expressed as an average quadratic cost and the plant is disturbed by Gaussian process and measurement noises. We establish several connections with previous works in the literature discussing, in particular, the relation to absolute and relative threshold policies. The usefulness of the results is illustrated through a numerical example.

2.1 Introduction

As mentioned in Chapter 1, recent research advocates that replacing periodic control and communication paradigms by event-triggered paradigms can have significant benefits in terms of reduced usage of communication, computation and energy resources. The fundamental idea behind event-triggered control (ETC) is that transmissions should be triggered by events inferred based on available state or output information, as opposed to being triggered periodically in time.

The pioneering works [30, 32, 77] proposed to trigger transmissions when the norm of the state or estimation error exceeds a certain threshold (absolute triggering). In particular, [77] showed that such a policy, also known as the absolute

[1]This chapter is based on [66].

threshold policy, can outperform periodic control in terms of a quadratic cost for the same average transmission rate for linear plants subject to Gaussian disturbances. Another influential work [33], considered a nonlinear model without disturbances and proposed to trigger transmissions in order to guarantee a decrease rate for a Lyapunov function. The resulting policy, often referred to as relative threshold policy, specifies that transmissions should occur when the norm of the error between the new and the previously sent data exceeds a weighted norm of the state. To combine the benefits of these two class of policies, in [44], mixed threshold policies were proposed, which is constructed by combining the relative and absolute threshold policies.

As an alternative, event-triggered control policies can also be obtained from optimal control formulations taking into account the closed-loop performance and the network usage (see, e.g., [34, 38–40, 46–51]). The closed-loop performance is typically defined in terms of a quadratic cost as in the celebrated LQR and LQG problems. Although the optimal event-triggered controller is in general computationally hard to find for these problems, some works following this approach have proposed suboptimal event-triggered controllers with guarantees on the closed-loop performance and/or on the network usage [26,40,46,48,59,60]. Interestingly, some of these suboptimal policies take the form of absolute [59] and relative [70] threshold policies, connecting well with the early works such as [77] and [33] as mentioned above.

However, such optimization-based methods typically assume that the full state feedback is available to schedule transmissions, whereas in many applications only partial (output) information is available for feedback. In fact, there appears to be no output-based event-triggered strategy with guaranteed closed-loop quadratic performance, although output-based strategies have been proposed in the context of other design approaches for event-triggered controllers (see, e.g., [36, 37, 44, 74, 78] and the references therein).

The contribution of the present work is to propose a new output-feedback controller, which is guaranteed to have a performance within a constant factor of the optimal periodic control performance (with all-time communication), while reducing the communication load. Performance is defined in terms of a quadratic average cost and we consider the co-design problem in which both the control inputs and the transmission decisions are designed simultaneously. The proposed transmission policy is based on a quadratic function of the state estimate obtained by the Kalman filter, while the control input is determined by a linear function of this state estimate. Several variants are discussed.

Interestingly, one of the proposed policies takes the form of a mixed threshold policy in which the absolute threshold term is a function of the steady-state estimation error covariance and the relative part depends on the state estimate. On the other hand, in the special case when no disturbances are present it boils down to the relative threshold policy provided in [70], which in turn is connected to the policy provided in [33].

We illustrate the usefulness of our results for the control of a system consisting of two masses and a spring using a communication network. The numerical results show that depending on the triggering mechanism approximately up to 70% communication reduction is achieved while guaranteeing a performance within 1.1 of the optimal periodic control performance (so only 10% performance loss).

The remainder of the chapter is organized as follows. In Section 2.2 we formulate the output-feedback ETC problem. Section 2.3 explains the proposed ETC method and provides its stability and performance guarantees. Section 2.4 presents the numerical example and Section 2.5 provides the conclusions.

Nomenclature

The trace of a square matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $\mathrm{Tr}(A)$. The expected value of a random vector η is denoted by $\mathbb{E}[\eta]$. For a symmetric matrix $Z \in \mathbb{R}^{n\times n}$, we write $Z \succ 0$ if Z is positive definite. For a symmetric matrix $X \in \mathbb{R}^{n\times n}$ we use for $x \in \mathbb{R}^n$ the notation $\|x\|_X^2 := x^T X x$ and $|s|$ represents the absolute value of the scalar $s \in \mathbb{R}$. Finally, we denote the set of nonnegative integers by $\mathbb{N}_0$.

2.2 Problem formulation

We consider the linear discrete-time system

$$\begin{aligned} x_{k+1} &= Ax_k + B\hat{u}_k + s_k \\ y_k &= Cx_k + v_k, \end{aligned} \tag{2.1}$$

where $x_k \in \mathbb{R}^{n_x}$, $\hat{u}_k \in \mathbb{R}^{n_u}$ and $y_k \in \mathbb{R}^{n_y}$ denote the state, the input, and the output, respectively, and s_k and v_k denote the state disturbance and the measurement noise, respectively, at discrete time $k \in \mathbb{N}_0$. We assume that $\{s_n\}_{n\in\mathbb{N}_0}$ and $\{v_n\}_{n\in\mathbb{N}_0}$ are sequences of zero-mean independent Gaussian random vectors with positive definite covariance matrices Φ_s and Φ_v, respectively. The initial state is assumed to be either a Gaussian random variable with mean $\bar{x}_0$ and covariance Θ_0 or known in which case $x_0 = \bar{x}_0$ and $\Theta_0 = 0$. Furthermore, we assume that (A, B) is controllable and (A, C) is observable.

We consider the performance measure

$$J = \limsup_{N\to\infty} \frac{1}{N}\, \mathbb{E}[\sum_{k=0}^{N-1} x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k + 2x_k^T S \hat{u}_k], \tag{2.2}$$

where Q, R, S are such that

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \succ 0. \tag{2.3}$$

We assume that a controller, collocated with the sensors, sends the control values to the actuators over a communication network. This controller should compute not only the control inputs, but should also decide at which times $k \in \mathbb{N}_0$ new control inputs are sent to the actuators. The setup is depicted in Figure 2.1-(a).

To model the occurrence of transmissions in the network, we introduce $\sigma_k \in \{0,1\},\ k \in \mathbb{N}_0$, as a decision variable such that $\sigma_k = 1$ indicates that a transmission occurs at time k and $\sigma_k = 0$ otherwise. We assume that at the actuator side a standard zero-order hold device is used as a control input generator (CIG) that holds the previous value of the control action if no new control input is received at time k (i.e. when $\sigma_k = 0$). We denote the computed (and transmitted) control value at time $k \in \mathbb{N}_0$ by u_k, when a transmission occurs, and any arbitrary value otherwise. Therefore, we have

$$\hat{u}_k = \begin{cases} \hat{u}_{k-1}, & \text{if } \sigma_k = 0, \\ u_k, & \text{if } \sigma_k = 1, \end{cases} \tag{2.4}$$

where $\hat{u}_{-1} := 0$. When $\sigma_k = 0$, we use the notation $u_k = \emptyset$ to indicate that the value of u_k is arbitrary (and actually irrelevant). This simple hold actuation mechanism is sufficient to illustrate the main ideas of the chapter. In Section 2.3.3 we consider an alternative model-based actuation mechanism to enhance the performance of our strategy even further.

Let I_k denote the information available to the controller at time $k \in \mathbb{N}_0$, i.e.,

$$I_k := \{y_0, \dots, y_k, u_0, \dots, u_{k-1}, \sigma_0, \dots, \sigma_{k-1}, \hat{x}_0, \Theta_0\}.$$

A policy $\pi := (\mu_0, \mu_1, \dots)$ is defined as a sequence of functions $\mu_k := (\mu_k^\sigma, \mu_k^u)$ that map the available information vector I_k into control actions u_k and scheduling decisions σ_k, $k \in \mathbb{N}_0$. We denote by J_π the cost (2.2) when policy π with

$$(\sigma_k, u_k) = \mu_k(I_k), \quad k \in \mathbb{N}_0, \tag{2.5}$$

is used. Moreover, we denote the average transmission rate as

$$R_t = \limsup_{N \to \infty} \frac{1}{N} \mathbb{E}[\sum_{k=0}^{N-1} \sigma_k]. \tag{2.6}$$

Ideally, we would like to find a policy π^* that minimizes the quadratic performance index (2.2) as well as the average transmission rate (2.6). This is a multi-objective mixed-integer average cost problem, which is in general hard to solve. Instead, we propose a policy π for which the cost J_π is within a constant factor of the corresponding cost $J_{\pi_{all}}$ of periodic (all-time) control π_{all} requiring a significantly smaller average transmission rate R_t.

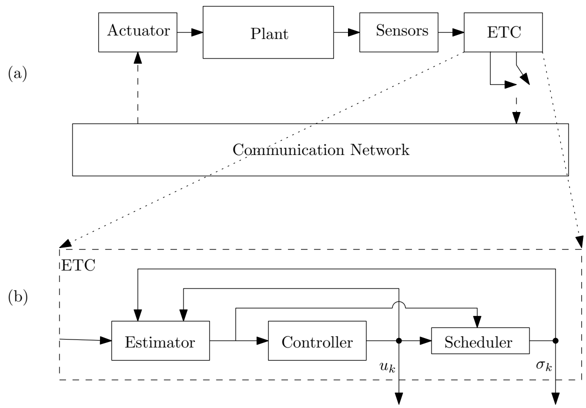


Figure 2.1. The schematic of the considered structure: (a) The overall loop structure: The sensors are connected to the ETC block, which decides the transmission times and computes the control input at the transmission instant. (b) Inside the ETC: A scheduler decides based on the available sufficient statistics provided by the estimator and the event generator (2.16) whether or not the computed control action will be transmitted.

2.3 Proposed method and main results

In Section 2.3.1 we present the proposed event-based controller and in Section 2.3.2 we provide our main result. In Section 2.3.3 we describe an alternative model-based actuation mechanism, and we state a similar result also for this case.

2.3.1 Proposed ETC method

The controller structure is shown in Figure 2.1-(b). The estimator computes an estimate of the plant's state based on the available information I_k, i.e., $\hat{x}_k := \mathbb{E}[x_k|I_k]$ which can be obtained by iterating the Kalman filter described for $k \in \mathbb{N}_0$ by

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + B\hat{u}_k + N_{k+1}z_{k+1} \\ z_{k+1} &= y_{k+1} - C(A\hat{x}_k + B\hat{u}_k) \end{aligned} \tag{2.7}$$

where $\hat{u}_k$ is defined as in (2.4), $\hat{x}_0 = \bar{x}_0 + N_0(y_0 - C\bar{x}_0)$ is the initial condition, and $N_s = \tilde{\Sigma}_s C^T(C\tilde{\Sigma}_s C^T + \Phi_v)^{-1}$ denotes the estimator gain. Where

$$\tilde{\Sigma}_{k+1} = \Phi_s + A\tilde{\Sigma}_k A^T - A\tilde{\Sigma}_k C^T(C\tilde{\Sigma}_k C^T + \Phi_v)^{-1}C\tilde{\Sigma}_k A^T \tag{2.8}$$

with initial condition $\tilde{\Sigma}_0 = \Theta_0$. Note that the estimation error covariance

$$\Sigma_k := \mathbb{E}\left[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T \middle| I_k\right]$$

can be described by (see, e.g., [71])

$$\Sigma_k = \tilde{\Sigma}_k - \tilde{\Sigma}_k C^T(C\tilde{\Sigma}_k C^T + \Phi_v)^{-1}C\tilde{\Sigma}_k. \tag{2.9}$$

The controller provides the input u_k to the plant only at transmission times, i.e., at times $k \in \mathbb{N}_0$ with $\sigma_k = 1$, and is described by a linear control function of the state estimate, taking the form

$$u_k = L\hat{x}_k, \tag{2.10}$$

where

$$\begin{aligned} L &= -(R + B^T K B)^{-1}(B^T K A + S^T) \\ K &= Q + A^T K A - P & \text{(2.11a)} \\ P &= (A^T K B + S)(R + B^T K B)^{-1}(B^T K A + S^T). & \text{(2.11b)} \end{aligned}$$

Note that there exists a unique positive definite solution K to the algebraic Riccati equation (2.11a)-(2.11b) due to our assumption that (A, B) is controllable and that (2.3) holds (see, e.g. [71]) and that the gain L coincides with the optimal gain for a state-feedback linear quadratic regulator with an all-time transmission policy π_{all} i.e. $(\sigma_k, u_k) = (1,\ L\hat{x}_k)$ for all $k \in \mathbb{N}_0$.

For the scheduler, we propose two event-triggered mechanisms (ETMs) for which we provide formal performance guarantees. The first ETM specifies that a transmission at time k occurs ($\sigma_k = 1$) if

$$\boxed{\|e_k\|_Z^2 > \|\hat{x}_k\|_Y^2 + \gamma,} \tag{2.12}$$

where $e_k := \hat{u}_{k-1} - L\hat{x}_k$ and $\gamma := \theta\,\mathrm{Tr}(Q\Sigma)$,

$$\begin{aligned} Y &:= \theta(Q + L^T R L + U - \epsilon I) \\ Z &:= R + (1+\theta)B^T K B + \frac{\theta}{\epsilon}(RL + S^T)(L^T R + S) \\ U &:= SL + L^T S^T, \end{aligned}$$

and ϵ is a given constant such that $0 < \epsilon < \lambda_{min}(Q + L^T R L + U)$ with λ_{min} denoting the smallest eigenvalue of the indicated matrix. Moreover, $\theta > 0$ is a tuning knob used to express the performance with respect to the all-time transmissions policy π_{all} (see (2.17) below). The second proposed ETM specifies that $\sigma_k = 1$ if

$$\boxed{\begin{bmatrix}\hat{x}_k^T & e_k^T\end{bmatrix}\Gamma\begin{bmatrix}\hat{x}_k \\ e_k\end{bmatrix} - \gamma > 0,} \tag{2.13}$$

with

$$\Gamma = \begin{bmatrix} -\theta(Q + L^T R L + U) & -\theta(L^T R + S) \\ -\theta(RL + S^T) & R + (1+\theta)B^T K B \end{bmatrix}. \tag{2.14}$$

Note that (2.13) has additional cross terms compared to (2.12), but, as we shall discuss bellow, will typically lead to less transmissions. The matrices and scalars in (2.12) and (2.13) are chosen in such a way that both ETMs will result in policies π for which we can guarantee a performance bound with respect to the optimal estimation and control corresponding to the all-time transmission policy π_{all}, i.e., $J_\pi \leq (1+\theta)J_{\pi_{all}}$. It is well known [71] that the optimal policy corresponding to the all-time transmission policy π_{all} is obtained by running the Kalman filter (2.7) for $\sigma_k = 1$, $k \in \mathbb{N}_0$, and computing the control law (2.10) for every $k \in \mathbb{N}_0$. This policy has a cost

$$J_{\pi_{all}} = \mathrm{Tr}(\Sigma Q) + \mathrm{Tr}\left(K(\tilde{\Sigma} - \Sigma)\right), \tag{2.15}$$

where $\Sigma = \lim_{s\to\infty} \Sigma_s$ and $\tilde{\Sigma} = \lim_{s\to\infty} \tilde{\Sigma}_s$.

2.3.2 Performance bounds

The main result of the chapter is presented next. We say that (2.1), (2.5) is mean square stable if $\sup_{k\in\mathbb{N}_0} \mathbb{E}[\|x_k\|^2] < \infty$ along all closed-loop trajectories.

Theorem 2.1. Consider system (2.1) with scheduling and control policy π defined as

$$(\sigma_k, u_k) = \begin{cases} \left(1, L\hat{x}_k\right), & \text{if (2.13) holds,} \\ \left(0, \emptyset\right), & \text{otherwise,} \end{cases} \tag{2.16}$$

where $\hat{x}_k$ is described by (2.7). Then the system (2.1), (2.5) is mean square stable for policy π and the associated average cost satisfies

$$J_\pi \leq (1+\theta)J_{\pi_{all}}. \tag{2.17}$$

Moreover, if we use (2.12) instead of (2.13) in (2.16), the same statements hold.

The parameter θ adjusts the trade-off between transmission rate R_t and the performance J_π. In fact, for $\theta = 0$, the transmissions are triggered when $e_k^T(R + B^TKB)e_k > 0$ which is satisfied if $e_k \neq 0$ and hence, always satisfied except for a set with zero measure. For this case, the proposed policies reduce (more or less) to the all-time transmission policy. On the other hand, to see the effect of large values of θ on the policies (2.12) and (2.13), we divide both sides of the policy inequalities by θ which lead to

$$\begin{aligned} \gamma &= \mathrm{Tr}(Q\Sigma) \\ Y &= (Q + L^TRL + U - \epsilon I) \\ Z &= \frac{1}{\theta}(R + B^TKB) + B^TKB + \frac{1}{\epsilon}(RL + S^T)(L^TR + S) \\ \Gamma &= \begin{bmatrix} -(Q + L^TRL + U) & -L^TR - S \\ -RL - S^T & \frac{1}{\theta}(R + B^TKB) + B^TKB \end{bmatrix}. \end{aligned}$$

As can be seen, increasing θ (enlarging the guaranteed bound) reduces the weight of e_k, and leads to less triggering in the sense that the sets $\{(\hat{x}_k,\, e_k) |\, (2.12)\ \textit{holds}\}$

and $\{(\hat{x}_k,\ e_k)|\ (2.13)\ holds\}$ become smaller and therefore increasing θ leads typically to less transmissions.

Before proving the theorem, several remarks are in order to establish connections between our result and existing works in the literature.

Remark 2.1. In the special case in which there is no process and measurement noise, and the full state is available for feedback ($C = I$), the Kalman filter estimate $\hat{x}_k$ coincides with the true state of the plant $x_k,\ k \in \mathbb{N}_0$. Moreover, in this case it holds that $\Sigma = 0$, $\gamma = 0$, and (2.12) boils down to

$$\|e_k\|_Z^2 > \|x_k\|_Y^2, \tag{2.18}$$

where now $e_k = \hat{u}_{k-1} - Lx_k,\ k \in \mathbb{N}_0$. This is a relative triggering policy in line with [33]. In addition (2.13) boils down to

$$\begin{bmatrix} x_k^T & e_k^T \end{bmatrix} \Gamma \begin{bmatrix} x_k \\ e_k, \end{bmatrix} > 0, \tag{2.19}$$

which can be shown to be equivalent to a policy provided in [70]. In [70] a similar bound on a deterministic performance index was obtained and the connection between such policy and the policy provided in [33] was established. Note that the present work extends [70] considering incomplete information and the presence of both process and measurement noises.

Remark 2.2. When the process and measurement noises are not zero, the triggering law (2.12) resembles that of a mixed ETM [44] characterized by Z, Y and γ. The absolute triggering part γ results from the uncertainty about the state due to the process and measurement disturbances. One can observe that the more uncertainty there is on the estimation showing itself in Σ and resulting in a larger γ, the more reluctant the schedulers are to transmit the data in the sense that the sets $\{(\hat{x}_k,\ e_k)|\ (2.12)\ holds\}$ and $\{(\hat{x}_k,\ e_k)|\ (2.13)\ holds\}$ are smaller.

Remark 2.3. Let $M := \theta(L^T R + S)$, $\theta > 0$ and $0 < \epsilon < \lambda_{min}(Q + L^T RL + U)$, then

$$0 \leq (\sqrt{\theta\epsilon}\hat{x} + \frac{1}{\sqrt{\theta\epsilon}}Me)^T(\sqrt{\theta\epsilon}\hat{x} + \frac{1}{\sqrt{\theta\epsilon}}Me) = \theta\epsilon\hat{x}^T\hat{x} + \frac{1}{\theta\epsilon}e^T M^T Me + 2\hat{x}^T Me.$$

This inequality implies that the cross term $-2\theta\hat{x}_k^T(L^T R + S)e_k$ in (2.13) is upper bounded by $\theta\epsilon\hat{x}^T\hat{x} + \frac{\theta}{\epsilon}e^T(RL + S)(L^T R + S^T)e$, which in turn results in

$$\begin{bmatrix} \hat{x}_k^T & e_k^T \end{bmatrix} \Gamma \begin{bmatrix} \hat{x}_k \\ e_k \end{bmatrix} \leq \begin{bmatrix} \hat{x}_k^T & e_k^T \end{bmatrix} \begin{bmatrix} -Y & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ e_k \end{bmatrix} = \|e_k\|_Z^2 - \|\hat{x}_k\|_Y^2. \tag{2.20}$$

That is, if for a given value of $\hat{x}_k$ and e_k at time $k \in \mathbb{N}_0$ (2.13) is satisfied, meaning that a transmission is triggered at time k using ETM (2.13), then (2.12) is also satisfied, but not vice-versa. Hence, (2.12) typically leads to more transmissions than (2.13).

Remark 2.4. Since (A, C) is observable, the discrete-time Riccati equation (2.8) converges to a steady-state solution $\tilde{\Sigma}$, i.e., $\tilde{\Sigma} = \lim_{s\to\infty} \tilde{\Sigma}_s$ as well. As a result Σ_k and N_{k+1}, $k \in \mathbb{N}_0$ converge to steady-state solutions $\Sigma := \lim_{s\to\infty} \Sigma_s$ and $N = \lim_{s\to\infty} N_s$ (see, e.g. [71]).

Proof of Theorem 2.1.

Note that

$$\mathbb{E}[(x_k - \hat{x}_k)\hat{x}_k^T | I_k] = 0, \quad k \in \mathbb{N}_0, \tag{2.21}$$

since

$$\mathbb{E}[(x_k - \hat{x}_k)\hat{x}_k^T | I_k] = (\mathbb{E}[x_k | I_k] - \hat{x}_k)\hat{x}_k^T = 0,$$

where we used the facts that given the information I_k, $\hat{x}_k$ is deterministic and $\hat{x}_k = \mathbb{E}[x_k|I_k]$. Therefore, for a given matrix X

$$\mathbb{E}[x_k^T X x_k | I_k] = \hat{x}_k^T X \hat{x}_k + \mathrm{Tr}(X\Sigma_k), \quad k \in \mathbb{N}_0. \tag{2.22}$$

Now suppose that we use the triggering policy (2.13) and the control policy (2.10) for every $k \in \mathbb{N}_0$. Then, if $\sigma_k = 1$, we have

$$\begin{aligned}
&\mathbb{E}[W(\hat{x}_{k+1}) + g(\xi_k, L\hat{x}_k, 1)|I_k] \overset{(2.7),(2.22)}{=} \\
&(1+\theta)\Big(\hat{x}_k^T(A^TKA + Q - P)\hat{x}_k + \mathrm{Tr}(\Sigma_k Q) + \mathrm{Tr}\left(K(\tilde{\Sigma}_{k+1} - \Sigma_k)\right)\Big) && (2.23)\\
&\le W(\hat{x}_k) + (1+\theta)\big(\mathrm{Tr}(\Sigma Q) + \mathrm{Tr}\left(K(\tilde{\Sigma} - \Sigma)\right) + \alpha_k\big), && (2.24)
\end{aligned}$$

where $\alpha_k = |\,\mathrm{Tr}\left(K\big((\tilde{\Sigma}_{k+1} - \tilde{\Sigma}) - (\Sigma_k - \Sigma)\big)\right)| + |\,\mathrm{Tr}((\Sigma_k - \Sigma)Q)|$. Note that in (2.23) we used the fact that

$$\mathbb{E}[z_{k+1}\hat{x}_k^T | I_k] = \mathbb{E}[(CA(x_k - \hat{x}_k) + Cs_k + v_k)\hat{x}_k^T | I_k] = 0,$$

due to (2.21), and $z_{k+1} \sim \mathcal{N}(0, C\tilde{\Sigma}_{k+1}C^T + \Phi_v)$ for all $k \in \mathbb{N}_0$, and the equality $\mathrm{Tr}(N_{k+1}^T K N_{k+1}(C\tilde{\Sigma}_{k+1}C^T + \Phi_v)) = \mathrm{Tr}(K(\tilde{\Sigma}_{k+1} - \Sigma_k))$. In turn, if $\sigma_k = 0$, it holds that

$$\begin{aligned}
&\mathbb{E}[W(\hat{x}_{k+1}) + g(\xi_k, \emptyset, 0)|I_k] \overset{(2.7),(2.22)}{=} W(\hat{x}_k) + e_k^T\big(R + (1+\theta)B^TKB\big)e_k \\
&\quad - \theta\big(\hat{x}_k^T(Q + L^TRL + U)\hat{x}_k + 2\hat{x}_k^T(L^TR + S)e_k + \mathrm{Tr}(\Sigma Q)\big) \\
&\quad + (1+\theta)\big(\mathrm{Tr}(\Sigma Q) + \mathrm{Tr}\left(K(\tilde{\Sigma} - \Sigma)\right) + \bar{\alpha}_k && (2.25)\\
&\overset{(2.13)}{<} W(\hat{x}_k) + (1+\theta)\big(\mathrm{Tr}(\Sigma Q) + \mathrm{Tr}\left(K(\tilde{\Sigma} - \Sigma)\right) + \alpha_k\big), && (2.26)
\end{aligned}$$

where $\bar{\alpha}_k = \mathrm{Tr}\left((\Sigma_k - \Sigma)Q\right) + (1+\theta)\,\mathrm{Tr}\left(K\big((\tilde{\Sigma}_{k+1} - \tilde{\Sigma}) - (\Sigma_k - \Sigma)\big)\right)$ and (2.25) follows by substituting $e_k + L\hat{x}_k$ for $\hat{u}_{k-1}$. We used the same facts to obtain (2.23) as the ones to obtain (2.25). Moreover, in (2.26) we used that $\bar{\alpha}_k \leq (1+\theta)\alpha_k$. Therefore, by combining (2.24) and (2.26) for the proposed policy π we have

$$\mathbb{E}[W(\hat{x}_{k+1}) + g(\xi_k, \mu^u(\hat{x}_k, e_k), \mu^\sigma(\hat{x}_k, e_k)) - W(\hat{x}_k)|I_k] \leq \\ (1+\theta)\Big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k\Big). \quad (2.27)$$

Adding (2.27) from $k = 0$ until $k = N-1$, and dividing by N, and conditioning over I_0, we obtain

$$\mathbb{E}\Big[\frac{1}{N}\sum_{k=0}^{N-1} \mathbb{E}[W(\hat{x}_{k+1}) - W(\hat{x}_k)|I_k]\big|I_0\Big] \\ + \mathbb{E}\Big[\frac{1}{N}\sum_{k=0}^{N-1} \mathbb{E}[g(\xi_k, \mu^u(\hat{x}_k, e_k), \mu^\sigma(\hat{x}_k, e_k))|I_k]\big|I_0\Big] \\ \leq (1+\theta)\frac{1}{N}\sum_{k=0}^{N-1}\big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k\big). \quad (2.28)$$

Using the tower property of conditional expectation, the first summation is given by $\frac{1}{N}(\mathbb{E}[W(\hat{x}_N) - W(\hat{x}_0))|I_0]$ and we shall prove that $\mathbb{E}[W(\hat{x}_N)|I_0]$ is bounded for all $N \in \mathbb{N}$. If we take $\limsup_{N\to\infty}$ on both sides of (2.28), the left-hand side is an upper bound of J_π and the right-hand side becomes the average cost of the all-time transmission policy π_{all} multiplied by $1+\theta$ from which we conclude (2.17). Note that $\limsup_{N\to\infty} \frac{1}{N}\sum_{k=0}^{N-1}\alpha_k = 0$ as $\lim_{k\to\infty}\alpha_k = 0$ based on the Stolz-Cesaro theorem.

We now prove that $\mathbb{E}[W(\hat{x}_N)|I_0]$ is bounded and that the system is mean square stable. Since $Q \succ 0$, we have

$$g(\xi, u, i) \geq x^T Q x \geq a_1 x^T x$$

for some positive constant a_1 and all $\xi \in \mathbb{R}^{n_\xi}$, $u \in \mathbb{R}^{n_u}$ and $i \in \{0, 1\}$. Hence,

$$\mathbb{E}[g(\xi_k, u_k, \sigma_k)|I_k] \overset{(2.22)}{\geq} a_1(\hat{x}_k^T\hat{x}_k + \operatorname{Tr}(\Sigma_k)).$$

Then, from (2.27) we conclude that

$$\mathbb{E}[\hat{x}_{k+1}^T K\hat{x}_{k+1} - \hat{x}_k^T K\hat{x}_k|I_k] \leq -\tilde{a}_1\hat{x}_k^T\hat{x}_k + d_1, \quad (2.29)$$

where $\tilde{a}_1 = \frac{a_1}{1+\theta}$ and d_1 is a positive constant such that

$$\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k \leq d_1 \quad \text{for all } k \in \mathbb{N}_0,$$

which exists since the estimation error covariance (2.9) remains bounded. Then

$$\mathbb{E}[\hat{x}_{k+1}^T K\hat{x}_{k+1}|I_k] \leq c_1\hat{x}_k^T K\hat{x}_k + d_1,$$

where $c_1 := 1 - \frac{\tilde{a}_1}{b_1}$ and b_1 is such that $K \preceq b_1 I$ and $1 - \frac{\tilde{a}_1}{b_1} < 1$. From this we conclude that (using again the tower property of conditional expectation)

$$\mathbb{E}[\hat{x}_N^T K \hat{x}_N | I_0] \leq c_1^N \hat{x}_0^T K \hat{x}_0 + \sum_{k=0}^{N-1} c_1^k d_1, \tag{2.30}$$

leading to the conclusion that $\mathbb{E}[\hat{x}_N^T K \hat{x}_N | I_0]$ is bounded as $N \to \infty$. Since K is positive definite, this leads to $\mathbb{E}[\hat{x}_N^T \hat{x}_N | I_0]$ being bounded as $N \to \infty$ and due to (2.22) so is $\mathbb{E}[x_N^T x_N | I_0]$, which shows mean square stability.

To prove that the theorem holds if the triggering mechanism (2.13) is replaced by (2.12), it suffices to observe that (2.27) holds also for the latter case. In fact, if (2.12) holds we have $\sigma_k = 1$ and we can follow the same steps as before concluding (2.24). On the other hand, if (2.12) does not hold ($\sigma_k = 0$) from Remark 2.3 we know that (2.13) would also not hold and then the same reasoning that led to (2.26) can be used. The same arguments can then be applied to establish the statement of the theorem for this case. □

Remark 2.5. Note that when the state and the noise covariances are zero, we can take $d_1 = 0$ and conclude from (2.30) and the fact that the state is then deterministic that the system is globally exponentially stable in a deterministic sense.

Remark 2.6. Parameter θ affects closed-loop performance through c_1 in (2.30). As θ increases, $\tilde{a}_1$ decreases, which results in the fact that c_1 approaches 1 and hence the bound on $\mathbb{E}[\|x_k\|^2]$ will increase.

Remark 2.7. The method we proposed can be perceived as a rollout procedure in the context of approximate dynamic programming (see, e.g., [71]) with the base policy π_{all}. Roughly speaking, $(1+\theta)W(\hat{x}_k)$ is employed as the approximation of the cost-to-go function in a one-step lookahead optimization, i.e.

$$J_{\pi_{stp}}(\hat{x}_k) = \min_{\sigma_k, u_k} \mathbb{E}[g(\xi_k, u_k, \sigma_k) + (1+\theta)W(\hat{x}_{k+1}) | I_k] \tag{2.31}$$

and the following policy can be shown to coincide with (2.16)

$$\mu_\pi(\hat{x}_k) = \arg\min_{\sigma_k, u_k} \mathbb{E}[g(\xi_k, u_k, \sigma_k) + (1+\theta)W(\hat{x}_{k+1}) | I_k], \tag{2.32}$$

where $J_{\pi_{stp}}(\hat{x}_k)$ is the one-step lookahead cost (see [62] for the discounted cost problem.)

2.3.3 A model-based actuation mechanism

An extension to the proposed ETC structure is to consider a model-based CIG [41], instead of the holding CIG, at the actuator side. Note that the corresponding implementation would require the availability of computational power at the actuator. The considered CIG is given by

$$\hat{u}_k = L\tilde{x}_k, \tag{2.33}$$

where

$$\tilde{x}_k = \begin{cases} (A+BL)\tilde{x}_{k-1}, & \sigma_k = 0 \\ \hat{x}_k, & \sigma_k = 1. \end{cases} \tag{2.34}$$

Theorem 2.2. Consider system (2.1) with scheduling policy

$$\sigma_k = \begin{cases} 1, & \text{if (2.13) holds with } e_k = \tilde{x}_k - \hat{x}_k \\ 0, & \text{otherwise} \end{cases} \tag{2.35}$$

and with

$$\Gamma = \begin{bmatrix} -\theta(Q + L^T RL + U) & -\theta(L^T R + S)L \\ -\theta L^T(RL + S^T) & L^T(R + (1+\theta)B^T KB)L \end{bmatrix} \tag{2.36}$$

and $\gamma = \theta \operatorname{Tr}(Q\Sigma)$. Then the system (2.1), (2.7), (2.33), (2.35) is mean square stable for the policy π_{mb} and the associated average stage cost satisfies

$$J_{\pi_{mb}} \leq (1+\theta)J_{\pi_{all}}, \tag{2.37}$$

where $J_{\pi_{mb}}$ is the cost of the policy (2.35), (2.33). Moreover, if we use (2.12) with $Y := \theta(Q + L^T RL + U - \epsilon I)$, $Z := L^T(R + (1+\theta)B^T KB + \frac{\theta}{\epsilon} RLL^T R)L$ with $0 < \epsilon < \lambda_{min}(Q + L^T RL + U)$ instead of (2.13) in (2.35), the same statements hold.

Proof. For $\xi := (x, \tilde{x}) \in \mathbb{R}^{2n_x}$, $j \in \{0, 1\}$, let $\bar{g}(\xi, j) = (1 + j\theta)(\xi^T \bar{Q} \xi)$ with

$$\bar{Q} := \begin{bmatrix} Q & SL \\ L^T S^T & L^T RL \end{bmatrix}$$

and define

$$W(\hat{x}_k) := (1+\theta)\hat{x}_k^T K \hat{x}_k.$$

Now suppose that we use the triggering policy (2.35) and the control policy (2.33) for every $k \in \mathbb{N}_0$. Then, if $\sigma_k = 1$

$$\begin{aligned} \mathbb{E}[W(\hat{x}_{k+1}) + \bar{g}(\xi_k, 1) | I_k] &\overset{(2.7),(2.22)}{=} (1+\theta)\Big(\hat{x}_k^T(A^T KA + Q - P)\hat{x}_k + \operatorname{Tr}(\Sigma_k Q) \\ &\quad + \operatorname{Tr}\big(K(\tilde{\Sigma}_{k+1} - \Sigma_k)\big)\Big) \qquad (2.38) \\ &\leq W(\hat{x}_k) + (1+\theta)\big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k\big), \qquad (2.39) \end{aligned}$$

where $\alpha_k = |\operatorname{Tr}\Big(K\big((\tilde{\Sigma}_{k+1} - \tilde{\Sigma}) - (\Sigma_k - \Sigma)\big)\Big)| + |\operatorname{Tr}((\Sigma_k - \Sigma)Q)|$. Note that in (2.38) we used the fact that $\mathbb{E}[z_{k+1}\hat{x}_k^T|I_k] = 0$, cf. (2.21), and

$$z_{k+1} \sim \mathcal{N}(0, C\tilde{\Sigma}_{k+1}C^T + \Phi_v)$$

for all $k \in \mathbb{N}_0$, and the equality $\operatorname{Tr}(N_{k+1}^T K N_{k+1}(C\tilde{\Sigma}_{k+1}C^T + \Phi_v)) = \operatorname{Tr}(K(\tilde{\Sigma}_{k+1} - \Sigma_k))$. In turn, if $\sigma_k = 0$, define $e_k := \tilde{x}_k - \hat{x}_k$, then

$$\begin{aligned}
&\mathbb{E}[W(\hat{x}_{k+1}) + \bar{g}(\xi_k, 0)|I_k] \\
&\overset{(2.7),(2.22)}{=} W(\hat{x}_k) + e_k^T L^T\big(R + (1+\theta)B^T K B\big)Le_k \\
&\quad - \theta\big(\hat{x}_k^T(Q + L^T RL + U)\hat{x}_k + 2\hat{x}_k^T(L^T R + S)Le_k \\
&\quad - \theta\operatorname{Tr}(\Sigma Q)\big) + (1+\theta)\big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) \\
&\quad - \theta\operatorname{Tr}\big((\Sigma_k - \Sigma)Q\big) + (1+\theta)\big(\operatorname{Tr}((\Sigma_k - \Sigma)Q) \\
&\quad + \operatorname{Tr}\Big(K\big((\tilde{\Sigma}_{k+1} - \tilde{\Sigma}) - (\Sigma_k - \Sigma)\big)\Big)\Big) &(2.40)\\
&\overset{(2.35)}{<} W(\hat{x}_k) + (1+\theta)\big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k\big), &(2.41)
\end{aligned}$$

where (2.40) follows by substituting $L(e_k + \hat{x}_k) = L\tilde{x}$ for $\hat{u}_k$ and the inequality (2.26) resulted from the triggering condition (2.35). Therefore, for the proposed policy π we have

$$\begin{aligned}
&\mathbb{E}[W(\hat{x}_{k+1}) + \bar{g}(\xi_k, \mu_{mb}^\sigma(\hat{x}_k, e_k)) - W(\hat{x}_k)|I_k] \leq \\
&\qquad (1+\theta)\Big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k\Big). &(2.42)
\end{aligned}$$

We used the same facts to obtain (2.38) as the ones to obtain (2.40).

Adding (2.42) from $k = 0$ until $k = N-1$, and dividing by N, and conditioning over I_0 we obtain

$$\begin{aligned}
&\mathbb{E}\Big[\frac{1}{N}\sum_{k=0}^{N-1}\mathbb{E}[W(\hat{x}_{k+1}) - W(\hat{x}_k)|I_k]\big|I_0\Big] + \mathbb{E}\Big[\frac{1}{N}\sum_{k=0}^{N-1}\mathbb{E}[\bar{g}(\xi_k, \mu_{mb}^\sigma(\hat{x}_k, e_k))|I_k]\big|I_0\Big] \\
&\qquad \leq (1+\theta)\frac{1}{N}\sum_{k=0}^{N-1}\big(\operatorname{Tr}(\Sigma Q) + \operatorname{Tr}\big(K(\tilde{\Sigma} - \Sigma)\big) + \alpha_k\big). &(2.43)
\end{aligned}$$

Using the tower property of conditional expectation, the first summation is given by given by $\frac{1}{N}(\mathbb{E}[W(\hat{x}_N) - W(\hat{x}_0))|I_0]$ and we shall prove that $\mathbb{E}[W(\hat{x}_N)|I_0]$ is bounded for all $N \in \mathbb{N}$. If we take $\limsup_{N\to\infty}$ on both sides of (2.28), the left-hand side is an upper bound of J_π and the right-hand side becomes the average cost of the all-time transmission policy π_{all} multiplied by $1+\theta$ from which we conclude (2.17). Note that $\limsup_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1}\alpha_k = 0$ as $\lim_{k\to\infty}\alpha_k = 0$ based on Stolz-Cesaro theorem.

We now prove that $\mathbb{E}[W(\hat{x}_N)|I_0]$ is bounded and that the system is mean square stable. Since $Q \succ 0$, we have

$$\bar{g}(\xi, i) \geq x^T Q x \geq a_1 x^T x$$

for some positive constant a_1 and all $\xi \in \mathbb{R}^{n_\xi}$, $u \in \mathbb{R}^{n_u}$ and $i \in \{0, 1\}$. Hence,

$$\mathbb{E}[\bar{g}(\xi_k, \sigma_k)|I_k] \overset{(2.22)}{\geq} a_1(\hat{x}_k^T \hat{x}_k + \mathrm{Tr}(\Sigma_k)).$$

Then, from (2.42) we conclude that

$$\mathbb{E}[\hat{x}_{k+1}^T K \hat{x}_{k+1} - \hat{x}_k^T K \hat{x}_k | I_k] \leq -\tilde{a}_1 \hat{x}_k^T \hat{x}_k + d_1, \tag{2.44}$$

where $\tilde{a}_1 = \frac{a_1}{1+\theta}$ and d_1 is a positive constant such that

$$\mathrm{Tr}(\Sigma Q) + \mathrm{Tr}\left(K(\tilde{\Sigma} - \Sigma)\right) + \alpha_k + \mathrm{Tr}(\Sigma_k) \leq d_1 \quad \text{for all } k \in \mathbb{N}_0$$

which exists since the estimation error covariance (2.9) remains bounded. Then

$$\mathbb{E}[\hat{x}_{k+1}^T K \hat{x}_{k+1} | I_k] \leq c_1 \hat{x}_k^T K \hat{x}_k + d_1,$$

where $c_1 := 1 - \frac{\tilde{a}_1}{b_1}$, b_1 is such that $K \preceq b_1 I$ and $1 - \frac{\tilde{a}_1}{b_1} < 1$. From this we conclude that

$$\mathbb{E}[\hat{x}_N^T K \hat{x}_N | I_0] \leq c_1^N \hat{x}_0^T K \hat{x}_0 + \sum_{k=0}^{N-1} c_1^k d_1, \tag{2.45}$$

leading to the conclusion that $\mathbb{E}[\hat{x}_N^T K \hat{x}_N | I_0]$ is bounded as $N \to \infty$. Since K is positive definite we also conclude that $\mathbb{E}[\hat{x}_N^T \hat{x}_N | I_0]$ is bounded as $N \to \infty$ and due to (2.22) so is $\mathbb{E}[x_N^T x_N | I_0]$, which shows mean square stability.

To prove that the theorem holds if the triggering mechanism (2.13) is replaced by (2.12), it suffices to observe that (2.42) holds also for the latter case. In fact, if (2.12) holds we have $\sigma_k = 1$ and we can follow the same steps as before concluding (2.39). On the other hand, if (2.12) does not hold ($\sigma_k = 0$) from Remark 2.3 we know that (2.13) would also not hold and then the same reasoning that led to (2.26) can be used. The same arguments can then be applied to establish the statement of the theorem. □

Clearly, it is expected (and illustrated by the example in the next section) that a model-based CIG can guarantee similar performance while reducing the communication even further. This is expected due to better control actions when there are no transmissions.

θ	$J_{\pi(2.12)}$	$J_{\pi(2.13)}$	R_t of (2.12)	R_t of (2.13)	$(1+\theta)J_{\pi_{all}}$
0.01	0.7397	0.7399	88%	84%	0.7408
0.1	0.7406	0.7500	80%	59%	0.8068
0.5	0.7408	0.7924	78.7%	38%	1.1002
1	0.7409	0.8292	78.5%	31%	1.4670
5	0.7409	0.9058	78.4%	23%	4.4009
10	0.7409	0.9244	78%	22%	8.0683

Table 2.1. Comparison of average cost and transmission rates of various value of θ for policies (2.12) and (2.13) with $J_{\pi_{all}} = 0.7335$.

2.4 Illustrative example

We consider an output-feedback version of the numerical example considered in [46]. This example consists of two unitary masses on a frictionless surface connected by an ideal spring with spring constant k_m and moving along a one-dimensional axis. The control input is a force acting on the first mass and the outputs are the position of both masses, i.e., x_1 and x_2. The state vector is $x = \begin{bmatrix} x_1 & x_2 & v_1 & v_2 \end{bmatrix}^T$, where v_1 and v_2 are the velocities of the masses. The equations of the process are

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c u_c \\ y_c &= C x_c, \end{aligned} \tag{2.46}$$

where

$$A_C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\kappa_m & \kappa_m & 0 & 0 \\ \kappa_m & -\kappa_m & 0 & 0 \end{bmatrix}, \quad B_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \tag{2.47}$$

By discretizing this system (for the continuous-time dynamics see [46]) with sampling period of $t_s = 0.1$ and using $k_m = 2\pi^2$, we obtain the model (2.46) with

$$A = \begin{bmatrix} 0.9045 & 0.0955 & 0.0968 & 0.0032 \\ 0.0955 & 0.9045 & 0.0032 & 0.0968 \\ -1.8466 & 1.8466 & 0.9045 & 0.0955 \\ 1.8 & -1.8 & 0.0955 & 0.9045 \end{bmatrix}, \; B = \begin{bmatrix} 0.0049 \\ 0.0001 \\ 0.0968 \\ 0.0032 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; \Phi_s = 0.01I, \; \Phi_v = 0.01I. \tag{2.48}$$

2.4.1 Comparison of the proposed policies

We consider an average cost problem with $Q = 0.1I$, $R = 0.1$ and $S = 0$. Table 2.1 summarizes the performance of the proposed policies (2.12) (with $\epsilon =$

CIG	R_t of (2.12)	R_t of (2.13)	$J_{\pi(2.12)}$	$J_{\pi(2.13)}$
ZOH	80%	59%	0.7406	0.7500
MB	50%	28%	0.7419	0.7523

Table 2.2. Comparison of the performance of theproposed policies (2.12) and (2.13) for model-based (MB) and zero-order hold (ZOH) CIGs with $\theta = 0.1$.

0.009) and (2.13) in comparison with the (periodic) all-time transmission policy π_{all} for various values of θ. In all cases, a time-varying Kalman filter is used to provide a state estimate based on the available information at each iteration. As expected, relaxing the performance requirements to $J_\pi \leq (1+\theta)J_{\pi_{all}}$ not only reduces the network usage significantly ranging from 12% to 78% reduction (depending on θ) compared to all-time transmission policy π_{all}, but also preserves the mean-square stability of the closed-loop system.

Moreover, we compare four cases applying the proposed policies with $\theta = 0.1$ using different CIGs in Table 2.2. As can be seen by using a model-based CIG an additional and significant reduction in network usage can be achieved with a similar average cost (computed through Monte Carlo simulation) compared to when a zero-order hold is used. It is also interesting to note that although choosing a large θ (e.g. $\theta = 10$) has a significant effect on the guaranteed bound, the actual cost is much less than the guaranteed bounds.

2.4.2 Comparison with periodic transmission policies

Although our proposed policy has guaranteed performance bounds, a valid question is to investigate how well the policy would perform compared to optimal periodic control policy characterized by sampling a continuous-time linear plant at equidistant time-interval, computing and transmitting the optimal control actions to the actuators which use a holding CIG between sampling times. In Figure 2.2 we compare the performances obtained with the optimal periodic control strategy and with the proposed ETC strategy (2.13) for several values of the average transmission period in the range [0.1, 0.33]. The parameter θ has been tuned to obtain the desired average transmission intervals and the cost has been computed via Monte Carlo simulation of 600 realizations for 600 time units and the noise characteristics are the same as in (2.48). For simulation purposes, we used the discretized version of the optimal control problem specified by the original continuous-time model and the cost

$$\lim_{T\to\infty} \frac{1}{T}\,\mathbb{E}\int_0^T \left[x_c(t)^T Q_c x_c(t) + u_c(t)^T R_c u_c(t)\right] dt, \tag{2.49}$$

where $Q_c = 0.05I,\ R_c = 0.05$. Assuming $u_c(t) = u_c(kt_s)$ for $t \in [kt_s,\ (k+1)t_s)$, we can obtain (2.1) and (2.2) in a similar manner as (2.48) for various values of t_s.

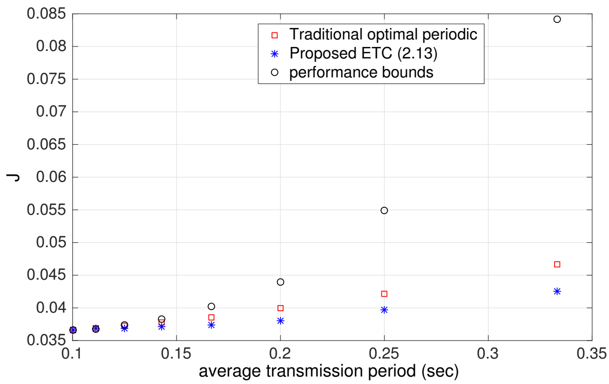


Figure 2.2. The comparison of the ETC mechanism (2.13) with the traditional periodic control strategy. The values of θ used for the ETC mechanism corresponding to the points in the figure from left to right were: $\theta \in \{0,\ 0.005,\ 0.02,\ 0.045,\ 0.1,\ 0.2,\ 0.5,\ 1.3\}$

The performance bounds are computed based on (2.17) where π_{all} corresponds to the periodic policy with $t_s = 0.1$. Note that for this simulation the corresponding values of Q, R, and S (in (2.2)) of the all-time transmission policy are obtained through the discretization of the cost (2.49) with sampling time $t_s = 0.1$. As can be seen for average transmission periods close to 0.1 (sec) the methods perform very closely. However, for larger transmission periods the proposed strategy (2.13) obtains significant performance improvements over traditional periodic control with the same average transmission period, which shows the advantage of the proposed method over the traditional periodic implementation.

2.5 Conclusions

In this chapter we proposed an optimization-based output-feedback event-triggered control solution for linear discrete-time systems with guaranteed performance expressed in terms of the optimal periodic (all-time) control performance, while reducing the communication load. The performance is measured by an average quadratic cost. Several connections with previous works in the literature have been established, and in particular, with the absolute, relative and mixed threshold policies [33, 44, 79]. The usefulness of the results was illustrated through a numerical example showing that a significant (up to 72%) reduction in network usage can be achieved by only sacrificing 10% of performance compared to the optimal all-time transmission policy. Furthermore, we showed that our proposed ETC can lead to significant improvements in the performance at the same average transmission rate when compared to optimal time-triggered *pe-*

riodic controllers. In Chapter 3, we will build upon the presented results to take into account multiple channels (i.e. also including a sensor-to-actuator controller network), where the objective is to design an scheduling mechanism for each individual channel while guaranteeing a performance bound.

Chapter 3

Remote sensing and control with performance guarantees

In this chapter[1], we consider a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The goal is to keep transmissions to a minimum while guaranteeing that the closed-loop performance is within acceptable bounds. This can be considered as an extension of the results of Chapter 2 taking into account a communication link between sensors and controller. The proposed policy in this new setting can be separated into an offline scheme for sensor query in which the transmission instances are computed a priori and an online scheme to schedule control input transmissions. The usefulness of the results is illustrated through two numerical examples.

3.1 Introduction

As mentioned in Chapter 1, the fundamental idea behind ETC is that transmissions should be triggered by events inferred from the state or the output of the plant. This in general leads to an improvement of the trade-off between average transmission rate and control performance when compared to periodic control, since in ETC the resources are used only when required.

Desirably, the communication protocols corresponding to ETC should still be insightful and simple to implement and guarantee some performance criteria for the control system. However, the analysis of most basic event-triggered schemes proposed in the literature focuses on fundamental system notions like Lyapunov stability, $\mathcal{L}_2$-norm gain guarantees and minimum inter-event time (see,

[1]This chapter is based on [63].

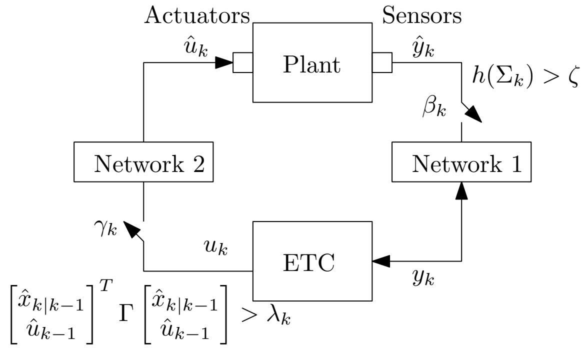


Figure 3.1. Overall setup and proposed policies: sensor query depends only on the Kalman filter covariance matrices Σ_k, while the control update transmissions are scheduled based on the Kalman filter state estimate $\hat{x}_{k|k-1}$, and the previously sent control input $\hat{u}_{k-1}$.

e.g., [33, 60, 72, 76, 80, 81]). These simple rules include transmitting only when a Lyapunov function would otherwise exceed a desired decay level (e.g., [33]), transmitting when the error between current measurements or control values and most recently sent ones exceeds an absolute or relative threshold (e.g. [74, 76]), or transmitting when the norm of the covariance of the Kalman filter exceeds a certain threshold (e.g. [72]). In turn, there are optimization-based schemes that guarantee closed-loop performance (see e.g., [26,38,46,48,49,59,60,77,82]). The performance is usually defined through a quadratic cost, as in the celebrated LQR problem, which is often combined with an additive or multiplicative penalty for transmissions. However, many of the proposed ETC policies following this approach are hard to implement in practice, and others lack the insight and simplicity of the basic policies just described.

The aim of this chapter is to provide simple ETC policies with guaranteed performance in terms of a quadratic cost. We consider a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The proposed ETC policies for sensor query and control input transmissions are derived using approximate dynamic programming (in particular, rollout techniques) for an optimization problem which includes a quadratic performance cost defined in terms of state and input variables and penalty terms for transmissions. The obtained main results extend our line of research presented in Chapter 2, where an event-triggered policy was designed only for scheduling control input transmissions, assuming that sensors measurements are available at every time instant. The new policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions. Both schemes are simple to implement and have insightful interpretation and thereby increasing the acceptance of practitioners (see Figure 3.1 for a first impression of which the details follow later).

In the online scheme determining the controller-to-actuator transmissions, the decisions are based on the state and control estimates, which are not known a priori and can be obtained via the time-varying Kalman filter. In the offline scheme for sensor query, transmissions are based on the covariances of the Kalman filter state estimates, which are known a priori. Interestingly, we can interpret this offline scheme as a policy in which transmissions occur when a function of the covariance of the Kalman filter exceeds a given threshold, which connects well to some policies proposed in this area of research (see, e.g., [72]).

The main advantage of our approach is that we can show that this event-triggered policy is stable (in a mean-square sense) and leads to performance guarantees in terms of the cost of all-time transmission policy, which was not the case in most other optimization-based ETC schemes. We discuss how this policy can be tuned to trade closed-loop performance guarantees and average transmission rates.

The usefulness of the results is illustrated through two numerical examples. The numerical results for both examples show that approximately 50% communication reduction is achieved while guaranteeing a performance bound when compared to periodic control implementations.

The remainder of the chapter is organized as follows. Section 3.2 formulates the problem. In Section 3.3 we state the main result of the chapter. Section 3.5 presents two examples. The first example consists of remote control of a scalar system and the second example is concerned with the control of double integrator over a communication network. Section 3.6 provides concluding remarks.

Nomenclature

The trace of a square matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $\mathrm{Tr}(A)$ and expected value vector and covariance matrix of a random $\eta \in \mathbb{R}^n$ are denoted by $\mathbb{E}[\eta]$ and $\mathrm{Cov}[\eta]$ respectively. For a symmetric matrix $Z \in \mathbb{R}^{n\times n}$, we write $Z \succ 0$ if Z is positive definite. The identity mapping is denoted by id and $\circ$ denotes composition operator.

3.2 Problem Formulation

We consider the remote control of a linear discrete-time system as depicted in Figure 3.1. The plant model is assumed to be given by

$$\begin{aligned} x_{k+1} &= Ax_k + B\hat{u}_k + v_k \\ \hat{y}_k &= Cx_k + r_k, \end{aligned} \tag{3.1}$$

where $x_k \in \mathbb{R}^{n_x}$, $\hat{u}_k \in \mathbb{R}^{n_u}$ and $\hat{y}_k \in \mathbb{R}^{n_y}$ denote the state, the input, and the output, respectively and v_k and r_k denote the state disturbance and measurement noise at time $k \in \mathbb{N}_0$. We assume that the disturbance and noise processes are Gaussian zero-mean, independent sequences of random vectors with covariances Φ_v and Φ_r, respectively. The initial state is assumed to be either a Gaussian random variable with mean $\hat{x}_0$ and covariance Θ_0 or known in which case x_0 equals $\hat{x}_0$ and $\Theta_0 = 0$.

The transmissions in the networks are modeled by introducing $\sigma_k = (\beta_k, \gamma_k) \in \{0,1\}^2, k \in \mathbb{N}_0$, as a decision vector where $\beta_k = 1$ (or $\gamma_k = 1$) indicates the occurrence of a transmission through the network from sensor to controller (or controller to actuator) at time k and $\beta_k = 0$ (or $\gamma_k = 0$) otherwise. We also consider that a standard zero-order hold device holds the most recently received value of the control action at the actuator side (in case no new control input is transmitted). Let u_k (or y_k) denote the sent (or received) value by the controller at time $k \in \mathbb{N}_0$ when a transmission occurs. We write $u_k = \emptyset$ (or $y_k = \emptyset$) to denote the case when at time $k \in \mathbb{N}_0$ no new values are transmitted.

The transmission decisions in both networks connecting the sensors to the controller and the controller to the actuators are assumed to be taken by the controller. For the network connecting the controller to the actuators, respectively, the controller needs only to send data when desired. However, for the network connecting the sensors to the controller, the controller must first query the sensors and then receive measurement data. We assume that the delay introduced by this process is negligible and assume there that there are no packet drops in both networks.

We consider the following cost to be minimized

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k)], \tag{3.2}$$

where $0 < \alpha < 1$ is the discount factor and $Q \succ 0, R \succ 0$ are proper (positive definite) weighting matrices. This cost is introduced for convenience as we are mostly interested in the minimization of the average cost defined as

$$\lim_{N\to\infty} \frac{1}{N} \mathbb{E}[\sum_{k=0}^{N-1} (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k)]. \tag{3.3}$$

Let I_{k-1} denote the information available to the controller at time k, i.e.,

$$I_{k-1} := (I_{k-2}, y_{k-1}, u_{k-1}, \sigma_{k-1})$$

for $k \in \mathbb{N}_{\geq 1}$ and $I_{-1} := (\hat{x}_0,\ \Theta_0)$. A policy $\pi := (\mu_0, \mu_1, \dots)$ is defined as a sequence of multivariate functions $\mu_k := (\mu_k^u, \mu_k^\sigma)$ that map the available information vector I_{k-1} into control actions u_k and scheduling σ_k, $k \in \mathbb{N}_0$.

We denote by $J_\pi^d(I_0)$ and J_π^a the costs (3.2) and (3.3), respectively, when

$$(u_k, \sigma_k) = \mu_k(I_{k-1}), \quad k \in \mathbb{N}_0. \tag{3.4}$$

We are interested in designing a policy that reduces the transmissions of the all-time transmission i.e. $\sigma_k = (1, 1)$ for every time step $k \in \mathbb{N}_0$, while keeping the performance within a desired bound of the performance of the all-time transmission policy. We recall that the optimal control policy corresponding to the all-time transmission is given by

$$\mu_{all}^{u}(I_{k-1}) = L\hat{x}_{k|k-1} \tag{3.5}$$

where

$$\begin{aligned} L &= -(R + \alpha B^T K B)^{-1} B^T K A \\ K &= Q + \alpha A^T K A - P \\ P &= \alpha^2 A^T K B (R + \alpha B^T K B)^{-1} B^T K A. \end{aligned} \tag{3.6}$$

and $\hat{x}_{k|k-1} := \mathbb{E}[x_k | I_{k-1}]$ can be obtained by running a time-varying Kalman filter [71]. This results in the following cost-to-go in the discounted case

$$J_{\pi_{all}}^{d}(I_{k-1}) = \mathbb{E}[x_k^T K x_k | I_{k-1}] + \frac{\alpha}{1-\alpha}\operatorname{Tr}(K\Phi_v) + \sum_{s=k}^{\infty} \alpha^{s-k} \operatorname{Tr}(P\Sigma_s), \tag{3.7}$$

where $\Sigma_s = \operatorname{Cov}[x_k | I_{k-1}]$ denotes the conditional covariance matrix of the estimation error that can be expressed as

$$\Sigma_s = \operatorname{Ric}^{s}(\Sigma_0), \quad s \in \mathbb{N}_0 \tag{3.8}$$

where

$$\operatorname{Ric}^{1}(\Sigma) = A\Sigma A^T + \Phi_v - A^T \Sigma C^T (C\Sigma C^T + \Phi_r)^{-1} C\Sigma A$$

and $\operatorname{Ric}^s = \operatorname{Ric}^{s-1} \circ \operatorname{Ric}^1$, $\operatorname{Ric}^0 = \mathrm{id}$. Furthermore, for the average cost problem the minimizing control policy is as (3.5) with $\alpha = 1$ and the corresponding average cost is

$$J_{\pi_{all}}^{a} := \operatorname{Tr}(K\Phi_v) + \operatorname{Tr}(P\bar{\Sigma}), \tag{3.9}$$

where $\bar{\Sigma}$ is the steady state covariance of the Kalman filter estimate defined as

$$\bar{\Sigma} = \lim_{s \to \infty} \Sigma_s.$$

and given by discrete Algebraic Riccati equation.

3.3 Main Result

The main result of the chapter is summarized in the following theorem. We say that (3.1) for a given policy π is mean square stable if $\sup_{k \in \mathbb{N}_0} \mathbb{E}[x_k^{\mathsf{T}} x_k] \leq a$ for some positive constant a.

Theorem 3.1. Consider system (3.1) with policy π_{ro} parameterized by two non-negative scalars ζ, θ and defined for $k \in \mathbb{N}_0$ by

$$(u_k, \gamma_k) = \begin{cases} (L\hat{x}_{k|k-1}, 1), & \text{if } \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{u}_{k-1} \end{bmatrix}^T \Gamma \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{u}_{k-1} \end{bmatrix} > \lambda_k \\ (\emptyset, 0), & \text{otherwise} \end{cases} \tag{3.10}$$

$$\beta_k = \begin{cases} 1, & \text{if } h(\Sigma_k) > \zeta \\ 0, & \text{otherwise,} \end{cases} \tag{3.11}$$

where

$$\begin{aligned} \Gamma &= \begin{bmatrix} (1+\theta)P - \theta Q & \alpha(1+\theta)A^T KB \\ \alpha(1+\theta)B^T KA & R + \alpha(1+\theta)B^T KB \end{bmatrix} \\ \lambda_k &= \theta \operatorname{Tr}(Q\Sigma_k) \\ \hat{x}_{k|k-1} &= \mathbb{E}[x_k|I_{k-1}] \\ \Sigma_k &= \operatorname{Cov}[x_k|I_{k-1}] \end{aligned} \tag{3.12}$$

and

$$h(\Sigma) = \sum_{s=0}^{\infty} \alpha^{s+1} \operatorname{Tr}\Big(P\big(\operatorname{Ric}^{s}(A\Sigma A^T + \Phi_v) - \operatorname{Ric}^{s+1}(\Sigma)\big)\Big) \tag{3.13}$$

is well defined in the sense that it is finite for every $\Sigma \in \mathbb{R}^{n_x \times n_x}$. Then

$$J^d_{\pi_{ro}}(I_0) \leq (1+\theta)(J^d_{\pi_{all}}(I_0) + \frac{1}{1-\alpha}\zeta), \tag{3.14}$$

for every I_0. Furthermore, for $\alpha = 1$, the system (3.1), (3.4) is mean square stable for policy π_{ro} and the associated average cost satisfies

$$J^a_{\pi_{ro}} \leq (1+\theta)(J^a_{\pi_{all}} + \zeta). \tag{3.15}$$

□

There are two options to compute $h(\Sigma)$. First, one can discretize the space of positive definite symmetric matrices of dimension $n_x \times n_x$ and compute beforehand the value of this function for a finite set of points Σ_i. Then, $h(\Sigma) \approx h(\Sigma_i)$ for a matrix Σ_i close to Σ. This is naturally only possible for small n_x. Second, we can approximate (3.13) by a finite summation (for a desirable precision), which must be computed online. The latter case requires more computational time and less memory resources than the former one.

We show in the state estimate subsection of current section that $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ and $\Sigma_k = \operatorname{Cov}[x_k|I_{k-1}]$ can be obtained by the controller by running the time-varying Kalman filter. As we shall see $\Sigma_k = \operatorname{Cov}[x_k|I_{k-1}]$ can be determined a priori, which entails that the scheduling sequence for sensor queries,

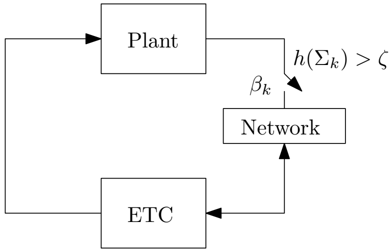


Figure 3.2. Sensor query schematic

triggered by condition (3.11) can be determined offline. In turn, the state estimate $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ depends on the noise realizations and therefore must be determined online. Consequently, the scheduling decisions, triggered by condition (3.10), must be determined by the controller online.

Remark 3.1. (Trade-off transmission versus performance guarantees) The two non-negative scalars θ and ζ should be seen as tunning knobs of the proposed method, which enable the adjustment of the transmission versus performance trade-off. It is clear that increasing ζ in (3.11) will make the sensor query triggering condition less stringent and thus one should expect less transmissions from sensors to the controller. This also leads to less tight performance guarantees (3.14),(3.15). It is also possible to see that increasing θ makes (3.10) less stringent and thus one should expect less transmissions from controller to actuators. Again, this also leads to less tight performance guarantees (3.14),(3.15). Note that if $\zeta = 0$ then transmissions from sensors to controller are triggered at every time step $k \in \mathbb{N}_0$, and if $\theta = 0$ then transmission from the controller to the actuators are triggered at every time step $k \in \mathbb{N}_0$. If $\zeta = 0$ and $\theta = 0$, we recover the all-time transmission control policy π_{all} and (3.14),(3.15) hold with equality.

Remark 3.2. When the controller is collocated with the actuators, as depicted in Figure 3.2, there is only one communication network. The scheduling variable β_k determines if a new measurement should be obtained or not according to the rule

$$\beta_k = \begin{cases} 1, & h(\Sigma_k) > \zeta \\ 0, & \text{otherwise} \end{cases}$$

and the control policy

$$\mu_k^{ro,u} = L\hat{x}_{k|k-1} \tag{3.16}$$

for $k \in \mathbb{N}_0$ that guarantees the performance bound of

$$J^d_{\pi_{ro}}(I_0) \leq J^d_{\pi_{all}}(I_0) + \frac{1}{1-\alpha}\zeta, \tag{3.17}$$

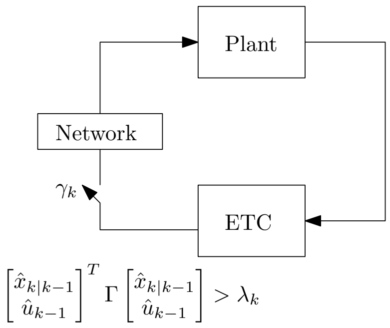


Figure 3.3. ETC collocated with the sensors

for every I_0 for the discounted cost. Furthermore, for $\alpha = 1$, the associated average cost satisfies

$$J^a_{\pi_{ro}} \leq J^a_{\pi_{all}} + \zeta. \tag{3.18}$$

Remark 3.3. Consider now the case where the controller is collocated with the sensors. This configuration, depicted in Figure 3.3, is similar to the one considered in Chapter 2, where a mixed triggering law with performance guarantees is proposed. The scheduling and control policies of Theorem 3.1, π_{ro}, become for $k \in \mathbb{N}_0$

$$\mu^{ro}_k = \begin{cases} \big(1, L\hat{x}_{k|k-1}\big), & \begin{bmatrix}\hat{x}_{k|k-1}\\ \hat{u}_{k-1}\end{bmatrix}^T \Gamma \begin{bmatrix}\hat{x}_{k|k-1}\\ \hat{u}_{k-1}\end{bmatrix} > \lambda_k \\ \big(0, \emptyset\big), & \text{otherwise,} \end{cases} \tag{3.19}$$

where

$$\begin{aligned} \Gamma &= \begin{bmatrix} (1+\theta)P - \theta Q & \alpha(1+\theta)A^TKB \\ \alpha(1+\theta)B^TKA & R + \alpha(1+\theta)B^TKB \end{bmatrix} \\ \lambda_k &= \theta \operatorname{Tr}(Q\Sigma_k). \end{aligned} \tag{3.20}$$

This policy guarantees the performance bound

$$J^d_{\pi_{ro}}(I_0) \leq (1+\theta) J^d_{\pi_{all}}(I_0), \tag{3.21}$$

for every I_0. Furthermore, for $\alpha = 1$, the associated average cost satisfies

$$J^a_{\pi_{ro}} \leq (1+\theta) J^a_{\pi_{all}}. \tag{3.22}$$

Hence, we recover here the results in Theorem 2.1 Chapter 2 as a special case of the general framework summarized in Theorem 3.1.

3.4 Derivation of the proposed ETC policy and proof of main results

3.4.1 Modeling as Switched Linear System

Considering again the setup in Section 3.2, the extended state $\xi_k := (x_k, \hat{u}_{k-1})$, we obtain the model

$$\begin{aligned} \xi_{k+1} &= A_{\gamma_k}\xi_k + B_{\gamma_k}u_k + \omega_k, \quad k \in \mathbb{N}_0 \\ y_k &= \begin{cases} \begin{bmatrix} C & 0 \end{bmatrix} \xi_k + r_k, & \beta_k = 1, \\ \emptyset, & \text{otherwise,} \end{cases} \end{aligned} \tag{3.23}$$

where $\omega_k := (v_k^T, 0_{n_u}^T)^T$ and

$$A_j := \begin{bmatrix} A & (1-j)B \\ 0 & (1-j)I \end{bmatrix},\ B_j := \begin{bmatrix} jB \\ jI \end{bmatrix},\ j \in \{0,1\}.$$

Moreover, the performance functions (3.2)-(3.3) can be written as

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k g(\xi_k, u_k, \gamma_k)], \tag{3.24}$$

$$\lim_{N\to\infty} \frac{1}{N}\,\mathbb{E}[\sum_{k=0}^{N-1} g(\xi_k, u_k, \gamma_k)], \tag{3.25}$$

where $g(\xi, u, j) = \xi^T Q_j \xi + u^T R_j u$ with

$$Q_j := \begin{bmatrix} Q & 0 \\ 0 & (1-j)R \end{bmatrix},\ R_j := jR,\ j \in \{0,1\}.$$

To penalize transmissions, we consider a multiplicative factor $f(\gamma_k)$ in the stage cost associated with time k defined as such

$$f(\gamma_k) = (1 + \theta\gamma_k) \quad \gamma_k \in \{0,1\} \quad \theta \geq 0, \tag{3.26}$$

and an additive term $c(\beta_k)$ defined as

$$c(\beta_k) = \delta\beta_k \quad \beta_k \in \{0,1\} \quad \delta \geq 0. \tag{3.27}$$

We consider the auxiliary cost functions described by

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k \big(f(\gamma_k) g(\xi_k, u_k, \gamma_k) + c(\beta_k)\big)], \tag{3.28}$$

$$\lim_{N\to\infty} \frac{1}{N}\,\mathbb{E}[\sum_{k=0}^{N-1} f(\gamma_k) g(\xi_k, u_k, \gamma_k) + c(\beta_k)]. \tag{3.29}$$

We denote by $V_\pi^d(I_0)$ and V_π^a the costs (3.28) and (3.29), respectively, when (3.4) is used. Our interest is to provide bounds on J_π^d, J_π^a as in (3.2), (3.3) by minimizing the costs V_π^d, V_π^a. The stated problem is an infinite horizon mixed integer programming problem, which is computationally intractable. As such, we propose a suboptimal approach based on limited lookahead policies and in particular on the rollout algorithm (see [71, pp. 304-307]) which has the ability to outperform all-time transmission control policy. In fact, we shall prove that for our proposed policies the costs are within bound of the corresponding costs using all-time transmission control policy while a reduction on average transmission rate in each network is achieved.

3.4.2 Rollout algorithm

In order to derive the proposed scheme, we use rollout procedure in the context of dynamic programming with the base policy π_{all} (3.5) corresponding to the case of all-time transmission i.e. $\sigma_k = (1,1)$ $\forall k \in \mathbb{N}_0$ that results in the following cost-to-go in the discounted case

$$V^d_{\pi_{all}}(I_{k-1}) = (1+\theta)\Bigg(\mathbb{E}\{x_k^T K x_k | I_{k-1}\} + \operatorname{Tr}(P\Sigma_k) + \frac{\alpha}{1-\alpha}\operatorname{Tr}(K\Phi_v) + \sum_{s=0}^{\infty} \alpha^{s+1} \operatorname{Tr}\left(P \operatorname{Ric}^{s+1}(\Sigma_k)\right)\Bigg) + \frac{1}{1-\alpha}\delta. \tag{3.30}$$

Moreover, for the average cost problem the minimizing control policy is as (3.5) with $\alpha = 1$ and the corresponding average cost is

$$V^a_{\pi_{all}} := (1+\theta)\big(\operatorname{Tr}(K\Phi_s) + \operatorname{Tr}(P\bar{\Sigma})\big) + \delta. \tag{3.31}$$

In fact, $V^d_{\pi_{all}}$ is employed as the approximation of the cost-to-go function defined as in (3.28) for a given information vector I_{k-1}, $k \in \mathbb{N}_0$, in a one-step lookahead optimization i.e.

$$V^d_{\pi_{stp}}(I_{k-1}) = \min_{u_k,\sigma_k} \mathbb{E}[\bar{g}(\xi_k, u_k, \sigma_k) + \alpha V^d_{\pi_{all}}(I_k) | I_{k-1}, u_k, \sigma_k] \tag{3.32}$$

where $\bar{g}(\xi, u, i, j) = (1 + i\theta) g(\xi, u, i) + j\delta$.

Therefore, at each iteration considering a limited lookahead policy of length one followed by an all-time transmission base policy there exist four possible scheduling policies as follows:

1. $\sigma_k = (0, 1)$. The controller does not inquire new measurement and computes the optimal control based on the available information. In this case the control policy is

$$\mu_k^u(I_{k-1}) = L\,\mathbb{E}\{x_k | I_{k-1}\} = L\hat{x}_{k|k-1} \tag{3.33}$$

and the Kalman filter works in the prediction mode i.e.

$$\begin{aligned} \hat{x}_{k+1|k} &= (A+BL)\hat{x}_{k|k-1} \\ \Sigma_{s+1} &= \operatorname{Ric}^{s-k}(A\Sigma_k A^T + \Phi_v), \quad s \geq k, \end{aligned} \tag{3.34}$$

therefore the one-step cost-to-go is

$$V^{d,(0,1)}(I_{k-1}) := (1+\theta)\Big(\mathbb{E}[x_k^T K x_k | I_{k-1}] + \operatorname{Tr}(P\Sigma_k) + \frac{\alpha}{1-\alpha}\operatorname{Tr}(K\Phi_v) + \sum_{s=0}^{\infty} \alpha^{s+1} \operatorname{Tr}\left(P \operatorname{Ric}^{s}(A\Sigma_k A^T + \Phi_v)\right)\Big) + \frac{\alpha}{1-\alpha}\delta \tag{3.35}$$

2. $\sigma_k = (1,0)$. The controller inquires new measurement but does not send new control action to the plant. The Kalman filter operates in the estimation mode i.e.

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k-1} + Bu_k + G_k\big(y_k - C\hat{x}_{k|k-1}\big) \\ G_k &= A\Sigma_k C^T (C\Sigma_k C^T + \Phi_r)^{-1} \\ \Sigma_{s+1} &= \operatorname{Ric}^{s+1-k}(\Sigma_k), \quad s \geq k, \end{aligned} \tag{3.36}$$

which results in the one-step cost-to-go

$$V^{d,(1,0)}(I_{k-1}) := \mathbb{E}\{x_k^T \bar{K} x_k | I_{k-1}\} + 2\alpha(1+\theta)\hat{x}_{k|k-1}^T A^T K B \hat{u}_{k-1} + (1+\theta)\Big(\frac{\alpha}{1-\alpha}\operatorname{Tr}(K\Phi_v) + \sum_{s=0}^{\infty}\alpha^{s+1}\operatorname{Tr}\left(P\operatorname{Ric}^{s+1}(\Sigma_k)\right)\Big) + \hat{u}_{k-1}^T \bar{R}\hat{u}_{k-1} + \frac{1}{1-\alpha}\delta, \tag{3.37}$$

where

$$\begin{aligned} \bar{R} &= R + \alpha(1+\theta)B^T K B \\ \bar{K} &= Q + \alpha(1+\theta)A^T K A. \end{aligned} \tag{3.38}$$

3. $\sigma_k = (0,0)$. There is no update in control action and also no inquiry for new information. Therefore, the Kalman filter operates in the prediction mode

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k-1} + B\hat{u}_{k-1} \\ \Sigma_{s+1} &= \operatorname{Ric}^{s-k}(A\Sigma_k A^T + \Phi_v), \quad s \geq k, \end{aligned}$$

therefore, the one-step cost-to-go is

$$V^{d,(0,0)}(I_{k-1}) := \mathbb{E}\{x_k^T \bar{K} x_k | I_{k-1}\} + 2\alpha(1+\theta)\hat{x}_{k|k-1}^T A^T K B \hat{u}_{k-1} + (1+\theta)\Big(\frac{\alpha}{1-\alpha}\operatorname{Tr}(K\Phi_v) + \sum_{s=0}^{\infty}\alpha^{s+1}\operatorname{Tr}\left(P\operatorname{Ric}^{s}(A\Sigma_k A^T + \Phi_v)\right)\Big) + \hat{u}_{k-1}^T \bar{R}\hat{u}_{k-1} + \frac{\alpha}{1-\alpha}\delta, \tag{3.39}$$

4. $\sigma_k = (1,1)$. The controller asks for new information and sends updated control actions to the plant. The one-step cost-to-go is the same as the all-time transmission (3.30) i.e.

$$V^{d,(1,1)}(I_{k-1}) = V^d_{\pi_{all}}(I_{k-1}). \tag{3.40}$$

Out of these four options, the one that has the least cost-to-go will determine the scheduling variables and corresponding control action. Therefore,

$$V^d_{\pi_{stp}}(I_{k-1}) = \min_{\sigma_k \in \{0,1\}^2} \{V^{d,\sigma_k}(I_{k-1})\} \tag{3.41}$$

or, in other words, the scheduling variable σ_k attains the minimum

$$\mu_k^{\sigma,ro}(I_{k-1}) = \arg\min_{\sigma_k} \{V^{d,\sigma_k}(I_{k-1})\}.$$

These comparison of the cost-to-goes lead to following results:

- Sensor query regardless of actuator update will result in an offline triggering rule for the sensor networks

$$V^{d,(0,1)} - V^{d,(1,1)} = V^{d,(0,0)} - V^{d,(1,0)} =$$
$$(1+\theta)\Big(\sum_{s=0}^{\infty} \alpha^{s+1} \operatorname{Tr}\big(P(\operatorname{Ric}^{s}(A\Sigma A^T + \Phi_v) - \operatorname{Ric}^{s+1}(\Sigma))\big) - \zeta\Big), \tag{3.42}$$

where $\zeta = \frac{\delta}{(1+\theta)}$.

- Actuation update regardless of the query from the sensors results in the dynamic triggering rule

$$V^{d,(1,0)} - V^{d,(1,1)} = V^{d,(0,0)} - V^{d,(0,1)} = \begin{bmatrix} \hat{x}^T_{k|k-1} & \hat{u}^T_{k-1} \end{bmatrix} \Gamma \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{u}_{k-1} \end{bmatrix} - \lambda_k \tag{3.43}$$

The interesting observation in here is that the triggering rule for the two networks are separated. The sensor query can be computed off-ine as in the case of [83,84]. The actuator network on the other hand has a dynamic triggering rule [62], see also Remark 3.2.

3.4.3 State Estimate

To obtain $\hat{x}_{k|k-1} = \mathbb{E}[x_k | I_{k-1}]$ and $\Sigma_k = \operatorname{Cov}[x_k | I_{k-1}]$ the controller can run the time-varying Kalman filter

$$
\begin{aligned}
\hat{x}_{k+1|k} &= A\hat{x}_{k|k-1} + Bu_k + \beta_k G_k\big(y_k - C\hat{x}_{k|k-1}\big) \\
G_k &= A\Sigma_k C^T(C\Sigma_k C^T + \Phi_r)^{-1} \\
\Sigma_{k+1} &= \overline{\mathrm{Ric}}(\Sigma_k, \beta_k),
\end{aligned} \tag{3.44}
$$

where

$$
\overline{\mathrm{Ric}}(\Sigma, j) = A\Sigma A^T + \Phi_v - jA^T\Sigma C^T(C\Sigma C^T + \Phi_r)^{-1}C\Sigma A, \tag{3.45}
$$

β_k is known and determined by the controller and u_k is the input to the plant, also known to the controller and defined by the recursion

$$
u_k = \begin{cases} u_{k-1}, & \text{if } \gamma_k = 0 \\ L\hat{x}_{k|k-1}, & \text{if } \gamma_k = 1. \end{cases}
$$

This follows from the fact that the conditional distribution of x_k given I_{k-1} is Gaussian (the proof of this fact can be concluded from a similar proof in [84]).

Remark 3.4. Note that since β_k is a function of Σ_k, given the initial error covariance Σ_k the error covariance evolves autonomously according to

$$
\Sigma_{k+1} = \overline{\mathrm{Ric}}(\Sigma_k, \beta_k(\Sigma_k)), k \in \mathbb{N}_0
$$

As a result Σ_k does not depend on the output noise and state disturbances realizations of the plant and can be determined a priori. In particular, the scheduling sequence of sensor transmission $\{\beta_k\}_{k\in\mathbb{N}_0}$ can be determined offline. In turn, the state estimate obtained from (3.44) depends on the output noise and state disturbances through y_k and therefore the corresponding Kalman filter iterations must be computed online. As a consequence, the scheduling sequence $\{\gamma_k\}_{k\in\mathbb{N}_0}$ determining control input transmission cannot be determined a priori.

3.4.4 Proof of Main Results

We consider first the discounted cost problem. For $\xi \in \mathbb{R}^{n_x+n_u}$, $u \in \mathbb{R}^{n_u}$, $i, j \in \{0, 1\}$, let $\bar{g}(\xi, u, i, j) = (1 + i\theta)g(\xi, u, i) + j\delta$. From (3.41) we conclude that

$$
\mathbb{E}[\alpha V^d_{\pi_{all}}(I_k) + \bar{g}(\xi_k, \mu_k^{u,ro}(I_{k-1}), \mu_k^{\sigma,ro}(I_{k-1}))|I_{k-1}] - V^d_{\pi_{all}}(I_{k-1}) \leq 0. \tag{3.46}
$$

for every $k \in \mathbb{N}_0$. Multiplying by α^k for each $k \in \mathbb{N}_0$, adding from $k = 0$ until $k = N - 1$, and conditioning over I_0 we obtain

$$
\begin{aligned}
&\mathbb{E}\Big[\sum_{k=0}^{N-1}\big(\alpha^{k+1}\mathbb{E}[V^d_{\pi_{all}}(I_k)|I_{k-1}] - \alpha^k V^d_{\pi_{all}}(I_{k-1})\big)|I_0\Big] \\
&\qquad + \mathbb{E}\Big[\sum_{k=0}^{N-1}\big(\mathbb{E}[\alpha^k\bar{g}(\xi_k, \mu_k^{u,ro}(I_{k-1}), \mu_k^{\sigma,ro}(I_{k-1}))|I_{k-1}]\big)|I_0\Big] \leq 0.
\end{aligned} \tag{3.47}
$$

Using the tower property of conditional expectation we have

$$\mathbb{E}\Big[\mathbb{E}[V^d_{\pi_{all}}(I_k)|I_{k-1}]|I_0\Big] = \mathbb{E}[V^d_{\pi_{all}}(I_k)|I_0]$$

from which we conclude that the first summation in (3.47) is a telescopic series and thus equal to

$$\mathbb{E}[\alpha^N V^d_{\pi_{all}}(I_N)|I_0] - V^d_{\pi_{all}}(I_0),$$

where $\alpha^N \mathbb{E}[V^d_{\pi_{all}}(I_N)|I_0] \to 0$ as $N \to \infty$. The second summation equals the cost of the rollout policy π_{ro} and thus we conclude

$$V^d_{\pi_{ro}}(I_0) \leq V^d_{\pi_{all}}(I_0).$$

It is clear that $J^d_{\pi_{ro}}(I_0) \leq V^d_{\pi_{ro}}(I_0)$ and $V^d_{\pi_{all}}(I_0) = (1+\theta)J^d_{\pi_{all}}(I_0) + \frac{1}{1-\alpha}\delta$ since V is obtained from J by multiplying by a factor always larger than or equal to 1, which is equal to $(1+\theta)$ in the case of all-time communication and adding the positive scalar $\frac{1}{1-\alpha}\delta$. Defining $\zeta := \frac{\delta}{1+\theta}$ results in (3.14).

We now consider the average cost problem. Consider that still $\alpha < 1$ and define

$$W(I_{k-1}) := (1+\theta)\,\mathbb{E}\{x_k^T K x_k | I_{k-1}\}.$$

From this definition and the definition of $V^d_{\pi_{all}}(I_{k-1})$ we conclude that

$$\begin{aligned}\mathbb{E}[\alpha W(I_k) + \bar{g}(\xi_k, \mu_k^{u,ro}(I_{k-1}), \mu_k^{\sigma,ro}(I_{k-1}))|I_{k-1}] - W(I_{k-1}) = \\ \mathbb{E}[\alpha V^d_{\pi_{all}}(I_k) + \bar{g}(\xi_k, \mu_k^{u,ro}(I_{k-1}), \mu_k^{\sigma,ro}(I_{k-1}))|I_{k-1}] - \\ V^d_{\pi_{all}}(I_{k-1}) + (1+\theta)\big(\alpha\,\mathrm{Tr}(K\Phi_v) + \mathrm{Tr}(P\Sigma_k)\big) + \delta. \end{aligned} \tag{3.48}$$

Using (3.46) we have

$$\begin{aligned}\mathbb{E}[\alpha W(I_k) + \bar{g}(\xi_k, \mu_k^{u,ro}(I_{k-1}), \mu_k^{\sigma,ro}(I_{k-1}))|I_{k-1}] - W(I_k) \leq \\ (1+\theta))\big(\alpha\,\mathrm{Tr}(K\Phi_v) + \mathrm{Tr}(P\Sigma_k)\big) + \delta. \end{aligned} \tag{3.49}$$

Making $\alpha = 1$ adding from $k = 0$ until $k = N-1$, dividing by N, and conditioning over I_0 we obtain

$$\begin{aligned}&\mathbb{E}\Big[\frac{1}{N}\sum_{k=0}^{N-1}\big(\mathbb{E}[W(I_k)|I_{k-1}] - W(I_k)\big)|I_0\Big] \\ &\quad + \mathbb{E}\Big[\frac{1}{N}\sum_{k=0}^{N-1}\big(\mathbb{E}[\bar{g}(\xi_k, \mu_k^{u,ro}(I_{k-1}), \mu_k^{\sigma,ro}(I_{k-1}))|I_{k-1}]\big)|I_0\Big] \\ &\qquad \leq (1+\theta)\frac{1}{N}\sum_{k=0}^{N-1}\big(\mathrm{Tr}(K\Phi_v) + \mathrm{Tr}(P\Sigma_k) + \zeta\big). \end{aligned} \tag{3.50}$$

Using again the tower property the first summation is given by

$$\frac{1}{N}(\mathbb{E}[W(I_N)|I_0] - W(I_0))$$

and we shall prove that $\mathbb{E}[W(I_N)|I_0]$ is bounded for every N. The second summation equals the average cost of the rollout policy $V^a_{\pi_{ro}}$ if we make $N \to \infty$ and the right-hand side is the average cost of the all-time transmission policy $V^a_{\pi_{all}}$. Then letting $N \to \infty$ we conclude that

$$V^a_{\pi_{ro}} \leq V^a_{\pi_{all}}$$

from which we can conclude (3.15).

It remains to prove that $\mathbb{E}[W(I_N)|I_0]$ is bounded and that the system is mean square stable. Note that for $\xi = (x, \hat{u}) \in \mathbb{R}^{n_x+n_u}$, $u \in \mathbb{R}^{n_u}$, $i, j \in \{0, 1\}$ we have

$$\bar{g}(\xi, u, i, j) \geq x^T Q x > a_1 x^T x$$

for some positive constant a_1. We can conclude that

$$\mathbb{E}[\bar{g}(\xi_k, u_k, \sigma_k)|I_{k-1}] > a_1\, \mathbb{E}[x^T x|I_{k-1}].$$

Then, from (3.49) we conclude that

$$\mathbb{E}[x^T_{k+1} K x_{k+1} - x^T_k K x_k|I_{k-1}] \leq -\tilde{a}_1\, \mathbb{E}[x^T x|I_{k-1}] + d_1$$

where $\tilde{a}_1 = \frac{a_1}{1+\theta}$ and d_1 is a positive constant such that

$$\mathrm{Tr}(P\Sigma^k_k) + Tr(K\Phi_\omega) \leq d_1 \quad \forall k \in \mathbb{N}_0$$

which exists since the covariance matrices of the Kalman filter iteration remain bounded. Then

$$\mathbb{E}[x^T_{k+1} K x_{k+1}|I_{k-1}] \leq c_1\, \mathbb{E}[x^T_k K x_k|I_{k-1}] + d_1$$

where $c_1 := 1 - \frac{a_1}{b_1}$, b_1 is such that $K < b_1 I$ and $1 - \frac{a_1}{b_1} < 1$. From this we conclude that

$$\mathbb{E}[x^T_N K x_N|I_0] \leq c_1^N \mathbb{E}[x^T_0 K x_0|I_0] + d_2,$$

leading to the conclusion that $\mathbb{E}[x^T_N K x_N|I_0]$ is bounded as $N \to \infty$. Since K is positive definite we also conclude that $\mathbb{E}[x^T_N x_N|I_0]$ is bounded as $N \to \infty$ which shows the mean square stability.

3.5 Numerical Examples

In this section, we present two numerical examples.

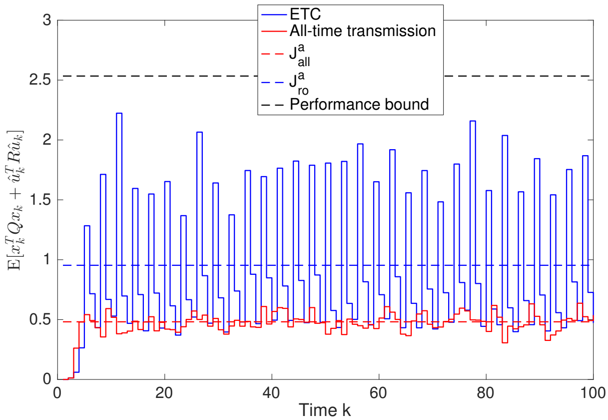


Figure 3.4. $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ estimated via Monte Carlo simulation in two cases with $\theta = \zeta = 0$ and $\theta = 0.2,\ \zeta = 1.65$.

3.5.1 Scalar system

Consider the scalar dynamical system

$$\begin{aligned} x_{k+1} &= 2x_k + \hat{u}_k + v_k \\ \hat{y}_k &= 3x_k + r_k, \end{aligned} \tag{3.51}$$

where v_k and r_k are Gaussian zero-mean, independent random sequences with covariance $\Phi_v = 0.01$ and $\Phi_r = 0.02$. The system is connected to the remote controller through two separate networks as depicted in Figure 3.1. We consider an average cost problem with $Q = 1$ and $R = 2$. We compare two cases, one with $\theta = \zeta = 0$ which corresponds to all-time transmission policy, π_{all}, and the other with $\theta = 0.2$ and $\zeta = 1.65$ implementing the triggering schemes, π_{ro}, (3.10)-(3.11). The method to compute the function h is based on discretization with a fine grid, as discussed directly after the formulation of Theorem 3.1. The state estimation and related covariances are computed using a time-varying Kalman filter. Interestingly, by applying the proposed ETC scheme, the sensor and actuator network usage reduce to 66% and 90%, respectively, while preserving the stability of the closed-loop system. Figure 3.4 shows the estimation of running cost $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ based on Monte Carlo simulations. As can be seen the average cost of the proposed algorithm satisfies the performance bound (3.15) as follow

$$0.97 = J^a_{\pi_{ro}} \leq 1.2(J^a_{\pi_{all}} + 1.63) = 2.53.$$

where $J^a_{\pi_{all}} = 0.4815$ computed using (3.9). Notice that the average cost of proposed ETC method $J_{\pi_{ro}}$ (blue dashed line in Figure 3.1) is much less than the theoretical performance bound (black dashed line in Figure 3.1) and close to the average cost of the all-time transmission policy $J^a_{\pi_{all}}$ (red dashed line

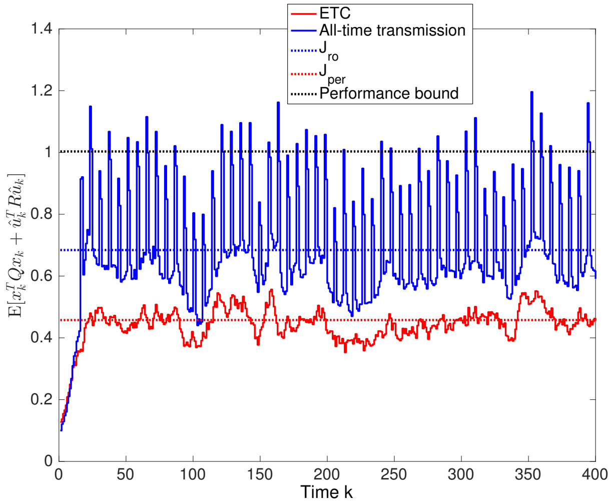


Figure 3.5. $\mathbb{E}[x_k^TQx_k + \hat{u}_k^TR\hat{u}_k]$ estimated via Monte Carlo simulation in two cases with $\theta = \zeta = 0$ and $\theta = 0.1,\ \zeta = 0.5$.

in Figure 3.1) which further emphasize that the reduction in network usage has been achieved with far less expected degradation in performance.

3.5.2 Double Integrator

We consider a discretized model of a double integrator controlled over network as depicted in Fig 3.1. The system and cost parameters are given by

$$A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad Q = 0.1I, \quad R = 0.01.$$

For this example, we compare two cases. The first one is with $\theta = 0,\ \zeta = 0$ i.e. the all-time transmission policy, π_{all}, and the other case based on π_{ro} as in Theorem 1 with $\theta = 0.1,\ \zeta = 0.5$. In both cases, a time-varying Kalman filter is used to provide a state estimation based on the available information at each iteration (i.e. the error covariance of the current state and the current state estimation). We compute h online using the method discussed right after Theorem 3.1. Figure 3.5 shows the estimated running cost $\mathbb{E}[x_k^TQx_k + \hat{u}_k^TR\hat{u}_k]$ for $k \in \mathbb{N}_0$ based on Monte Carlo simulations in both cases. The blue and red dotted lines represent $J^a_{\pi_{ro}}$ and $J^a_{\pi_{all}}$ respectively. The black dotted line shows the theoretical bounds computed using (3.9) and (3.15) which satisfies

$$0.6842 = J^a_{\pi_{ro}} \leq 1.1(J^a_{\pi_{all}} + 0.5) = 1.05.$$

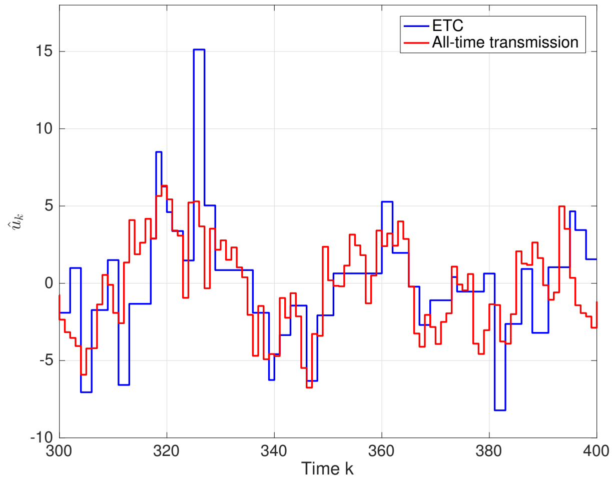


Figure 3.6. Trajectory of actuation signal, $\hat{u}_k$, for two cases with $\theta = \zeta = 0$ and $\theta = 0.1,\ \zeta = 0.5$.

where $J^a_{\pi_{all}} = 0.45$. Similar to previous example, one can observe that the theoretical upper bound (black dotted line in Figure 3.5) is not tight and the average cost of the proposed ETC scheme $J^a_{\pi_{ro}}$ (blue dotted line in Figure 3.5) is much closer to the corresponding average cost of all-time transmission control policy $J^a_{\pi_{all}}$ (red dotted line in Figure 3.5). Interestingly, the proposed ETC scheme not only reduces the network usage to 14% at the sensor side and 9% at the actuator side with respect to the all-time transmission case but also preserves the stability of the closed-loop system while guaranteeing a performance bound. To see the reduction in communications, the actuator signals of one realization of both cases with the same noise are shown in Figure 3.6. As can be seen, the actuator holds its value more often (i.e. less transmissions occur) when applying the proposed method.

3.6 Conclusions

In this chapter we proposed a simple and easy to implement ETC strategy for linear discrete-time with performance guarantees compared to the all-time transmission control policy, while reducing the overall communication load. The considered setup consists of a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. Our proposed approach is obtained via an optimization-based scheme in which the performance is measured by a quadratic cost. The resulting policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions.

The usefulness of the results was illustrated through two numerical examples that led to significant communication savings with only a moderate loss in performance compared to the all-time transmission policy. In Chapter 6 we examine the usefulness of the proposed algorithm in an experimental setup to remotely regulate a ground robot toward the origin in 1D.

Part II

Consistent event-triggered controller

Chapter 4

A consistent dynamic event-triggered policy for linear quadratic control

In this chapter[1], we define two desired consistency properties for ETC: (i) it should result in a better trade-off between average transmission rate and closed-loop performance than traditional periodic control; (ii) it should require no sensor updates (i.e., operate in open loop) in the absence of disturbances. We propose an ETC for linear systems with full state feedback that guarantees these two properties when performance is measured by an average quadratic cost. However, we show via an example that these properties are not necessarily met for threshold-based policies for which transmissions are triggered if the euclidean norm of the error between the current state and the previously transmitted state is less that a threshold.

4.1 Introduction

Although the literature on ETC is by now quite vast, there is limited concern about guaranteeing some natural desirable properties underlying the basic concept of ETC. In this chapter we propose two such properties and provide an event-triggered scheme that meets these. The first property is to achieve a better trade-off between average transmission rate and closed-loop performance than traditional periodic control; the second is to require no sensor updates (i.e., operate in open loop) in the absence of disturbances. We say that an ETC is consistent if it satisfies these two properties. Note that, the second property

[1]This chapter is based on [67, 70].

only holds for ETC strategies in which a control-input-generator (CIG) based on system model is implemented in the actuator side and the full state (not the output) of the system is transmitted via the network. Clearly in a control loop, where a zero-order hold (ZOH) is used as the CIG, the second property can not be guaranteed for an ETC. Interestingly, in the context of Lyapunov-based ETC design, the work [85] mentions a similar property to first consistency property as a general requirement for any ETC scheme.

Many well-know ETC methods do not satisfy at least one of these properties. In [46] a policy is provided that guarantees the first consistency property for any discrete-time linear system perturbed by additive white Gaussian noise and considering quadratic discounted and average costs to assess performance. Yet, even if the disturbances are arbitrarily small or absent such a scheme still transmits at a large rate, and therefore does not satisfy the second consistency property. The work [41] meets the second consistency property by using a state estimator at the actuators side and triggering transmissions only if the state disturbances make the state estimation error higher than a threshold. However, the first consistency property is not necessarily met, since performance is not addressed in [41]. The idea to alternate between using a fast and a slow periodic sampling scheme proposed in [48] is promising and leads to a better trade-off between average transmission rate and a quadratic performance index than periodic control. However, to avoid generating transmissions (or generate as few as possible) in the absence of disturbances, the slow sampling scheme would have to correspond to a very large sampling period, affecting also performance significantly. The ETC method proposed in [73], can be seen as a prelude to consistent ETC, since it satisfies the two consistency properties, but it is only useful when the disturbances are sporadic; otherwise it yields similar transmission patterns to periodic control. In general, it is hard to find a policy that assures that the consistent properties are satisfied in a given performance sense in other well-known works on ETC [33, 34, 47, 81].

Our novel ETC is designed for linear continuous-time systems with disturbances. As proposed in [73], disturbances are modeled as state jumps at times spaced by stochastic exponentially distributed intervals; this disturbance model is quite broad and, as we shall discuss below, it can also capture more traditional additive white Gaussian noise models [77]. Performance is measured by an average quadratic cost, as in the standard linear quadratic gaussian (LQG) framework. The proposed solution builds upon a key result and on the trade-off curve between average sampling period and average quadratic cost performance for periodic control. The key result states that if full state-feedback is available to determine transmission times, then the considered average cost can be written in terms of a cost that depends only on the state of the process between two consecutive transmissions. The proposed event-triggered method is then designed to minimize this latter cost and in particular to yield a better cost than periodic control for the same average transmission rate. This is illustrated for a double integrator process. Moreover, it assures that no transmissions occur in the absence of disturbances. This is therefore a consistent policy. While it is not hard to find an example where the second property of consistency does not hold, the effectiveness of ETC policies make it nontrivial to find an example when ETC policies perform worse than periodic control. However, we manage to provide an example of a linear quadratic control problem for which a traditional ETC policy, where transmissions occur if the Euclidean norm of the error between the system's state and a state estimate exceeds a threshold, does not satisfy the first consistency property.

The remainder of the chapter is organized as follows. Section 4.2 provides the problem formulation. The two consistency properties are introduced in this section. Section 4.3 establishes two key results and Section 4.4 describes the proposed ETC method establishing that it is consistent. Section 4.5 presents simulation results and Section 4.6 provides concluding remarks.

4.2 Problem Formulation

We consider the following linear model for the process to be controlled

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad t \in \mathbb{R}_{\geq 0} \backslash \mathcal{E}, \tag{4.1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state and $u(t) \in \mathbb{R}^{n_u}$ is the control input, (A, B) is assumed to be controllable, and the probability distribution of the initial condition x_0 is denoted by μ_0, with mean $\mathbb{E}[x_0] = \bar{x}_0$ and finite covariance; $\mathcal{E} := \{s_\ell\}_{\ell \in \mathbb{N}}$ is a set of event times at which the state undergoes a jump modeled by

$$x(s_\ell) = x(s_\ell^-) + w_\ell, \tag{4.2}$$

where $\{w_\ell\}_{\ell \in \mathbb{N}}$ is a sequence of independent and identically distributed random vectors with zero mean and finite covariance. We denote by μ the probability measure of w_ℓ, i.e., $\text{Prob}[w_\ell \in E] = \mu(E)$ for any open set E and for every $\ell \in \mathbb{N}$ and denote by $W := \mathbb{E}[w_\ell w_\ell^\top]$ the covariance of w_ℓ. We assume that $\mu(\{0\}) = \text{Prob}[w_k = 0] < 1$. Besides the state jump times $0 < s_1 < s_2 < \dots$, we define $s_0 = 0$ and assume that the time intervals $\{b_{\ell+1} := s_{\ell+1} - s_\ell\}_{\ell \in \mathbb{N}_0}$, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, are independent and exponentially distributed with rate λ, i.e., $\text{Prob}[s_{\ell+1} - s_\ell > a] = e^{-\lambda a}$, for every $\ell \in \mathbb{N}_0$. The sample space of the underlying probability space will be assumed to be $\Omega = \mathcal{A}$, where

$$\mathcal{A} := \{\omega = (x_0, \omega_0) | x_0 \in \mathbb{R}^n, \omega_0 = ((b_1, w_1), (b_2, w_2), \dots), b_i \in \mathbb{R}_{\geq 0}, w_i \in \mathbb{R}^n\}, \tag{4.3}$$

and the probability measure is such that the component x_0 of $\omega = (x_0, \omega_0) \in \Omega$ is distributed according to μ_0 and each component (b_i, w_i) of ω_0 is such that b_i is an exponentially distributed random variable with rate λ and w_i is a random vector with distribution μ.

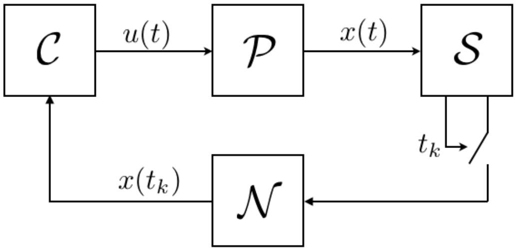


Figure 4.1. Considered ETC setup; $\mathcal{P}$-plant; $\mathcal{C}$-controller; $\mathcal{N}$-network; $\mathcal{S}$-scheduler

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This model is a special case of the model proposed in [73], which can capture several important applications of ETC. In particular, it can capture the more commonly used model in the event-triggered community (see [77])

$$d\underline{x} = (\underline{A}\underline{x} + \underline{B}u)dt + B_w d\underline{w}, \tag{4.4}$$

where $\underline{w}$ is an n_x-dimensional Wiener process with incremental covariance $I_n dt$ (the generalized mean square derivative is white Gaussian noise). This can be achieved by a construction similar to the one described in [75, Sec.3.2], [73]. However, the considered model is broader, and in particular can model cases where the disturbances occur sporadically, by making λ small [73].

We assume that data transmissions between the plant sensors and the actuators should be kept to a minimum. For instance, this is important when there is a communication network with limited bandwidth connecting sensor and actuators. The ETC consists of two distributed components, a scheduler and a controller, on the sensor and actuator sides, respectively, as depicted in Figure. 4.1.

The scheduler determines when sensor data should be transmitted to the controller/actuators based on the information provided by the sensor measurements. We assume that the sensors provide full state information and denote the transmissions times by $\{t_k\}_{k\in\mathbb{N}}$ and $t_0 := 0$. Formally, the transmission times to be specified by the scheduler are stopping times $0 < t_1 < t_2 < \dots$, which, by definition (see, e.g., [86, Ch. 1]), are random variables $t_k : \Omega \to \mathbb{R}_{\geq 0}$ such that the event $[t_k \leq t]$ belongs to the natural filtration of the process (4.1), (4.2) up to time t. Intuitively, given the state information up to time t, $\mathcal{I}^s(t) := \{x(r)|r \in [0,t]\}$, one can decide if t_k has occurred before t or not. At times t_k the scheduler sends $x(t_k)$ to the controller. Motivated by this fact, we restrict the class of policies for the transmission times to take the form

$$t_{k+1} = t_k + \tau_k,$$

where τ_k is the inter-transmission time between two consecutive transmissions.

We restrict the scheduler policy to be such that $\mathbb{E}[\tau_k] < \infty$, for every k, which is always the case if the inter-transmission times τ_k are bounded. A scheduler is then specified by the sequence of times

$$\zeta = \{\tau_0, \tau_1, \tau_2, \dots\}.$$

The controller determines the control input applied by the actuators to the plant. Although the controller only receives state measurements at transmission times, it can in general actuate the plant in continuous-time. Hence, the controller must decide $u(t) = \mu(t, \mathcal{I}^c(t))$ based on the information set $\mathcal{I}^c(t) := \{x(t_k)|t_k \leq t\}$, where $\mu(t, .)$ is in general a time-dependent control policy.

An ETC, denoted by π, is then specified by a control policy and a scheduling policy

$$\pi = \{\mu(t, .), \zeta\}. \tag{4.5}$$

Performance is measured by the following average cost

$$J := \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}[\int_0^T x(t)^\top Qx(t) + u(t)^\top Ru(t)dt], \tag{4.6}$$

for positive definite matrices Q and R, which is typically considered in the context of linear quadratic control. The average inter-transmission time associated with a given scheduler and controller policy is $\limsup_{T\to\infty} \frac{1}{T}\mathbb{E}[\sum_{k=0}^{\infty} 1_{t_k\leq T}]$, where $1_{t_k \leq T} = 1$ if $t_k \leq T$ and $1_{t_k\leq T} = 0$ if $t_k > T$; the average inter-transmission time is

$$\bar{\tau} := \limsup_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} \mathbb{E}[\tau_k]. \tag{4.7}$$

Naturally, the average transmission rate equals $1/\bar{\tau}$. In the case of periodic control, the scheduler is fixed and specified by $t_k = kh$, where h is the sampling period and coincides with the average inter-transmission time $\bar{\tau} = h$. As we shall see shortly, given this periodic scheduler, we can obtain the optimal controller. The resulting optimal average cost is denoted by $J_{\text{per}}(h)$.

Definition 4.1. Let J_π and $\bar{\tau}_\pi$ denote the average quadratic cost and the average inter-transmission time of an event-triggered policy π, defined in (4.5). We say that an ETC policy π is *consistent* if:

- $J_\pi < J_{\text{per}}(\bar{\tau}_\pi)$, i.e., if the event-triggered policy achieves a better performance than that of periodic control for the same average inter-transmission time (or average transmission rate).
- $t_{k+1} \geq \bar{s}_k$, for every $k \in \mathbb{N}_0$, where

$$\bar{s}_k := \min\{s_\ell | s_\ell > t_k, \ell \in \mathbb{N}\}, \tag{4.8}$$

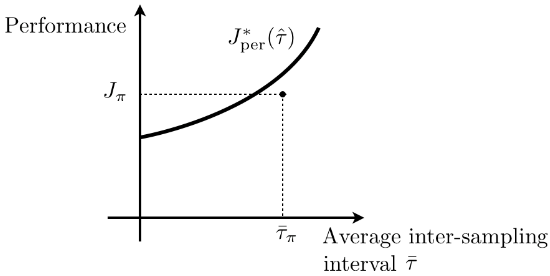


Figure 4.2. Illustration of the first consistent property: for the same average transmission rate $\bar{\tau}$, the performance of the ETC J_π is better than that of periodic control $J_{\mathrm{per}}(\bar{\tau}_\pi)$.

i.e., the scheduler generates no transmission after a given time t_k when no disturbances occur between the previous transmission time t_k and the current time t.

In practice, it is convenient to bound the inter-transmission time by a large constant T. We say that an ETC policy is T-consistent if the first consistency property is satisfied and if $t_{k+1} \geq \min\{\bar{s}_k, t_k + T\}$, for every $k \in \mathbb{N}_0$ and for a fixed T

$\square$

The problem considered in this chapter is to find a consistent policy.

4.3 Two key results

The proposed ETC builds upon two key results presented in this section. To obtain the first result we fix the scheduler to periodic transmissions and design the optimal controller. To obtain the second result we fix the controller, which takes the same form of the optimal periodic controller, and rewrite the average cost (4.6) achieved by the event-triggered policy consisting of this controller and a special class of schedulers.

The first result is provided next and it is a special case of [73, Ths. 2 and 3]. Let $\mathrm{tr}(M)$ denote the trace of a square matrix M.

Theorem 4.1. Suppose that the scheduler corresponds to periodic transmissions with period $h \in \mathbb{R}_{>0}$, i.e., $t_k = kh, \quad k \in \mathbb{N}_0$. Then the control policy that minimizes the average cost (4.6) is given by

$$u(t) = K\hat{x}(t),$$

where

$$K := -R^{-1}B^\top P, \tag{4.9}$$

P is the unique positive definite solution to

$$A^\top P + PA - PBR^{-1}B^\top P + Q = 0 \tag{4.10}$$

and $\hat{x}(t)$ is provided by the following state estimator

$$\begin{aligned} \dot{\hat{x}}(t) &= (A + BK)\hat{x}(t),\ t \in \mathbb{R}_{\geq 0} \backslash \{kh\}_{k \in \mathbb{N}_0}, \\ \hat{x}(kh) &= x(kh), \quad k \in \mathbb{N}_0, \quad \hat{x}(0) = \bar{x}_0. \end{aligned} \tag{4.11}$$

Moreover, the optimal cost is given by

$$J_{\text{per}}(h) = \text{tr}(P(\lambda W)) + g(h),$$

where

$$g(h) = \begin{cases} \dfrac{1}{h}\text{tr}(K^\top RK \displaystyle\int_0^h V(s)ds), \text{ if } h > 0, \\ 0, \text{ otherwise} \end{cases} \tag{4.12}$$

and $V(s) = \int_0^s e^{Ar}(\lambda W)e^{A^\intercal r}dr$.

□

The theorem is a special case of [73, Ths. 2 and 3] and the proof can be found in [87].

As discussed in [73], for a given $\tau \geq 0$, the matrix $V(\tau)$ corresponds to the covariance of the error

$$e(t) := x(t) - \hat{x}(t) \tag{4.13}$$

at time $t_k + \tau$, i.e. $V(\tau) = \mathbb{E}[e(t_k + \tau)e(t_k + \tau)^\intercal]$ for any $k \in \mathbb{N}_0$. Note that the control policy depends in fact on the previously transmitted states.

This result shows that the optimal average cost of the periodic control strategy is given by the sum of a constant term $\text{tr}(P\lambda W)$ and a term $g(h)$ that depends on the sampling period h. Moreover, it is insightful to notice that as h approaches zero the performance approaches $\text{tr}(P\lambda W)$, which is the optimal performance when the full state $x(t)$ is available (cf. [73]).

The added cost term $g(h)$ results from the mismatch between the true state $x(t)$ and the state estimate $\hat{x}(t)$, which the controller obtains by running (4.11). Note that given λ, K, R, and A, we can obtain an analytical expression for this function in terms of the sum of exponentials and polynomials (divided by h). In particular, it is a differentiable function. Moreover, the following holds.

Lemma 4.1. The function g described by (4.12) is non-decreasing for every $h \in \mathbb{R}_{\geq 0}$ and the right derivative at 0, $\lim_{h \to 0, h > 0} \frac{g(h) - g(0)}{h}$, equals $\frac{1}{2}tr(RK(\lambda W)K^\intercal)$ and is positive if $KW \neq 0$.

□

Proof. Note that we can write g as

$$g(h) = \frac{1}{h}\int_0^h z(s)ds, \quad z(s) := \mathrm{tr}(RKV(s)K^\intercal).$$

It is clear that $V(s)$ is positive definite for every $s \in \mathbb{R}_{>0}$ and $V(t) \geq V(s)$ if $t > s$. Then $z(s)$ is non-decreasing and since $g(h)$ is the mean value of an non-decreasing function in the interval $[0, h]$ it is also non-decreasing. Moreover, if we expand $V(s)$ in Taylor series around $s = 0$ we obtain $V(s) = sW + \text{h.o.t}$ where h.o.t denotes higher order terms. Then $g(h) = \frac{1}{h}tr(RK(\lambda W)K^\intercal)\frac{h^2}{2} + \text{h.o.t}$ and since R is positive definite, a sufficient condition for the derivative of $g(s)$ at zero to be different from zero is that $KW \neq 0$. □

The two properties stated in this lemma express the intuitive fact that the cost of the optimal periodic control policy increases (or at least does not decrease) with the period, and will play a key role in obtaining a consistent event-triggered policy.

To state the second key result, we restrict the controller to take the same form as that of the optimal control law for periodic control

$$\begin{aligned} u(t) &= K\hat{x}(t) \\ \dot{\hat{x}}(t) &= (A + BK)\hat{x}(t),\ t \in \mathbb{R}_{\geq 0}\backslash\{t_k\}_{k\in\mathbb{N}_0}, \\ \hat{x}(t_k) &= x(t_k), \quad k \in \mathbb{N}_0, \quad \hat{x}(0) = \bar{x}_0. \end{aligned} \tag{4.14}$$

However, the transmission intervals $t_{k+1}-t_k$, $k \in \mathbb{N}_0$ are not necessarily constant, as in periodic control, but are determined by the τ_k. Moreover, we restrict the set of stopping policies with the help of a function $\theta : \mathcal{A} \to \mathbb{R}_{\geq 0}$ coinciding with the first inter-transmission time τ_0, i.e.,

$$\tau_0(\omega) = \theta(\omega) = \theta(x_0, (b_1, w_1), (b_2, w_2), \dots). \tag{4.15}$$

Intuitively, the other inter-transmission times τ_k are defined exactly as τ_0 but for the process restarted at time t_k, i.e., $\{x(t_k + r)|r \in \mathbb{R}_{\geq 0}\}$ (see, for instance, the class of schedulers proposed in (4.30) below). Formally we have, for $k \in \mathbb{N}$,

$$\tau_k(\omega) = \theta(x(t_k), (\bar{b}_{\underline{\ell}(t_k)}, w_{\underline{\ell}(t_k)}), (b_{\underline{\ell}(t_k)+1}, w_{\underline{\ell}(t_k)+1}), \dots)), \tag{4.16}$$

where

$$\underline{\ell}(t_k) := \{\min \ell | s_\ell \geq t_k\}$$

and $\bar{b}_{\underline{\ell}(t_k)} := s_{\underline{\ell}(t_k)} - t_k$ is also exponentially distributed with rate λ, due to the memoryless property of the exponential distribution. Note that (4.16) also holds for $k = 0$, in which case it coincides with (4.15).

Theorem 4.2. Consider an ETC with control policy (4.14) and scheduling policy (4.15), (4.16). Then, the cost (4.6) can be written as

$$J = g_{\mathrm{ETC}} + \mathrm{tr}(P(\lambda W)), \tag{4.17}$$

where

$$g_{\mathrm{ETC}} := \frac{1}{\mathbb{E}[\theta(\omega)]}\mathbb{E}[\int_0^{\theta(\omega)} e(s)^\intercal K^\intercal R K e(s) ds], \tag{4.18}$$

provided that the expectations in (4.17) are finite.

□

Note that the expectations in (4.17) are finite if the inter-transmission times τ_k are bounded.

Proof. In [73, Th.1] it is shown for a broader class of models than (4.1), (4.2) that the cost (4.6) can be decomposed into the sum of a constant and a term that depends on the difference between the control input $u(t)$ and the optimal continuous-time controller $K\hat{x}(t)$. For the special case considered in the present work, we have

$$J = \mathrm{tr}(P\lambda W) + \lim_{T\to\infty}\frac{1}{T}\mathbb{E}[\int_0^T (u(s) - Kx(s))^\intercal R(u(s) - Kx(s))]ds \tag{4.19}$$

where K and P are defined in (4.9), (4.10). This is a standard result in optimal control (see, e.g. [88, Lemma 6.1, Ch. 8] adapted in [73, Th.1] to the model (4.1), (4.2). Since the controller is fixed and given by (4.14), we have

$$u(t) - Kx(t) = -Ke(t).$$

Then, if we let $N(T) := \max\{k \in \mathbb{N}_0 | t_k < T\}$ we have

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\mathbb{E}[\int_0^T (u(t)-Kx(t))^\intercal R(u(t)-Kx(t))] = \lim_{T\to\infty}\frac{1}{T}\mathbb{E}[\int_0^T e(t)^\intercal K^\intercal R K e(t)] \\
&= \lim_{T\to\infty}\frac{1}{T}\mathbb{E}[\sum_{k=0}^{N(T)-1}\int_{t_k}^{t_{k+1}} e(t)^\intercal K^\intercal R K e(t)dt] + \frac{1}{T}\mathbb{E}[\int_{t_{N(T)}}^T e(t)^\intercal K^\intercal R K e(t)dt]
\end{aligned} \tag{4.20}$$

Note that $e(t)$ is described by

$$\begin{aligned}
\dot{e}(t) &= Ae(t), \quad t \in \mathbb{R} \setminus (\mathcal{E} \cup \{t_k | k \in \mathbb{N}_0\}) \\
e(s_\ell) &= e(s_\ell^-) + w_\ell, \quad \ell \in \mathbb{N}_0, \\
e(t_k) &= 0, \quad k \in \mathbb{N}_0,
\end{aligned} \tag{4.21}$$

for an initial condition $e(0) = 0$. Due to the memoryless property of the exponential distribution we have that

$$y_k := \int_{t_k}^{t_{k+1}} e(t)^\intercal K^\intercal R K e(t) dt \tag{4.22}$$

are independent and identically distributed random variables with finite expectation by assumption. By the same token $t_{k+1} - t_k$ are independent and identically distributed random variables with expectation $\mathbb{E}[\theta_\alpha]$, which is finite by assumption. Then $\sum_{k=0}^{N(T)-1} y_k$ is a renewal process as defined [89, Sec. 3.4] and from [89, Prop. 3.41] (which builds upon the strong law of large numbers) we concluded that

$$\lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{N(T)-1} y_k = \frac{\mathbb{E}[y_0]}{\mathbb{E}[t_1 - t_0]}, \tag{4.23}$$

which implies that

$$\mathbb{E}[\sum_{k=0}^{N(T)-1} \int_{t_k}^{t_{k+1}} e(t)^\intercal K^\intercal R K e(t) dt = \frac{1}{\mathbb{E}[\theta_\alpha]} \mathbb{E}[\int_0^\tau e(t)^\intercal K^\intercal R K e(t) dt]. \tag{4.24}$$

Taking (4.24) into account while taking the limit for the first term in (4.20), noticing that the last term in (4.20) converges to zero, and recalling (4.19) we conclude (4.17). □

A special case of the scheduler (4.15), (4.16) is a periodic scheduler with $\theta(\omega) = h$ for every $\omega \in \Omega$, in which case (4.17) boils down to (4.12), since in such a case $\mathbb{E}[\theta(\omega)] = h$, and

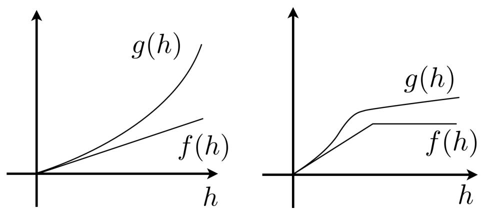
$$\mathbb{E}[\int_0^\theta e(s)^\intercal K^\intercal R K e(s) ds] = \int_0^h \operatorname{tr}(K^\intercal R K \mathbb{E}[e(s)e(s)^\intercal]) ds,$$

where as mentioned before $\mathbb{E}[e(s)e(s)^\intercal] = V(s)$ for $s \in [0, h]$. However, the cost contribution in (4.17) introduced by a general scheduling scheme of the ETC, g_{ETC}, can be minimized by choosing an adequate scheduler different from periodic, as we show in the next section.

4.4 Consistent ETC method and main results

The proposed ETC method builds upon the cost $g(h)$ of the periodic control strategy. Hereafter, we assume $KW \neq 0$. Let $f(h)$ be a non-decreasing continuous function defined in a given interval $h \in [\underline{h}, \bar{h}]$, $\underline{h} \in \mathbb{R}_{\geq} 0$ and $\bar{h} \in \mathbb{R}_{>0} \cup \{\infty\}$, such that

$$f(h) < g(h), \text{ for } h \in (\underline{h}, \bar{h}). \tag{4.25}$$



(a) convex g (b) arbitrary g

Figure 4.3. Illustration of two special functions $f < g$. When g is convex f can be linear, otherwise it can saturate after a given $h = \epsilon$

We impose that f is positive in the interval $h \in [\underline{h}, \bar{h}]$, except when $\underline{h} = 0$, in which case $f(0) = 0$. An important special case is when $\underline{h} = 0$ and

$$f(h) = Ch, \quad h \in [0, \bar{h}) \tag{4.26}$$

for some positive C. In particular, due to Lemma 4.1, if g is convex we can pick $C \in (0, \frac{1}{2}\mathrm{tr}(RK(\lambda W)K^\intercal))$ and (4.25) holds. If g is not convex, we can pick

$$f(t) = \begin{cases} Ct, & \text{if } t \in [0, \epsilon), \\ C\epsilon & \text{if } t \in [\epsilon, \bar{h}), \end{cases} \tag{4.27}$$

where due to Lemma 4.1 we can always find positive scalars C and ϵ such that (4.25) holds. The two cases are illustrated in Figure 4.3. In both cases $\bar{h}$ might be bounded or $\bar{h} = \infty$.

Given f, we start by defining a class of scheduling policies taking the form (4.15), (4.16), characterized by a family of functions θ_α parameterized by a positive scalar α

$$\theta_\alpha(\omega) = \inf \left\{ \beta \in [\underline{h}, \bar{h}] \,\middle|\, \eta_\alpha(\beta) \geq f(\beta) \right\}, \tag{4.28}$$

where

$$\eta_\alpha(\beta) := \frac{1}{\alpha} \int_0^\beta e(s)^\intercal K^\intercal R K e(s) ds. \tag{4.29}$$

The corresponding scheduling policy is

$$\tau_k = \inf\{\beta_k \in [\underline{h}, \bar{h}] | \frac{1}{\alpha} \int_0^{\beta_k} e(t_k + s)^\intercal K^\intercal R K e(t_k + s) ds \geq f(\beta_k)\}. \tag{4.30}$$

The proposed ETC policy is defined for α such that

$$\alpha \leq \mathbb{E}[\theta_\alpha(\omega)]. \tag{4.31}$$

Note that these policies are well defined in the sense that the minimum in (4.30) is always achieved. This follows from the fact that the error is a piecewise continuous function for every realization of the random variables and the integral of a quadratic function of the error is then continuous which is also the case for f by construction. Moreover, note that the error $e(t)$ (see (4.21)) resets to zero at each transmission time t_k and due to the memoryless property of the exponential distribution, the first disturbance time after each t_k has the same distribution of that of the first disturbance time after the initial time $t_0 = 0$. Therefore, we can conclude that

$$\mathbb{E}[\tau_k] = \mathbb{E}[\theta_\alpha], \text{ for every } k \in \mathbb{N}_0.$$

The next lemma allows us to conclude that we can always find α such that (4.31) holds.

Lemma 4.2. Let

$$L(\alpha) := \mathbb{E}[\theta_\alpha(\omega)].$$

Then:

(i) if $\underline{h} > 0$, then $L(\underline{h}) > \underline{h}$.

(ii) if $\underline{h} = 0$, then there is $\epsilon > 0$ such that $L(\epsilon) > \epsilon$.

(iii) if $\bar{h} < \infty$, then $L(\bar{h}) < \bar{h}$.

(iv) $L(\alpha)$ is a non-decreasing function of $\alpha > 0$.

□

Proof. Let $b_\ell := s_{\ell+1} - s_\ell$, for every $\ell \in \mathbb{N}_0$ and let

$$\Omega = \{w_\ell, b_\ell | \ell \in \mathbb{N}_0\} \tag{4.32}$$

be the basic probability space. Consider the following partition $\Omega = \Omega_1 \cup \Omega_2$ where

$$\Omega_1 := \{\omega \in \Omega | \theta_\alpha(\omega) = \underline{h}\}, \quad \Omega_2 := \{\omega \in \Omega | \theta_\alpha(\omega) > \underline{h}\} \tag{4.33}$$

for $\alpha = \underline{h}$. In Ω_2 lies for example the event $[s_1 > \underline{h}] = [b_0 > \underline{h}]$ which has non-zero probability. It is then clear that $\text{Prob}[\Omega_2] > 0$. Thus

$$\begin{aligned} L(\alpha)_{|\alpha=\underline{h}} &= \mathbb{E}[\theta_\alpha(\omega)] = \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_1}] + \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_2}] \\ &= \underline{h}\text{Prob}[\Omega_1] + \mathbb{E}[\theta_\alpha(\omega)|\omega \in \mathbf{1}_{\Omega_2}]\text{Prob}[\Omega_2] > \underline{h}. \end{aligned}$$

To prove (ii) we define the following sets, for a given $\alpha > 0$ and a given $\epsilon > 0$, $\Omega_1^\epsilon := \{\omega \in \Omega | \theta_\alpha(\omega) \geq \epsilon\}$, $\Omega_2^\epsilon := \{\omega \in \Omega | \theta_\alpha(\omega) < \epsilon\}$. Then,

$$\begin{aligned} L(\alpha) = \mathbb{E}[\theta_\alpha(\omega)] &= \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_1^\epsilon}] + \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_2^\epsilon}] \\ &\geq \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_1^\epsilon}] \geq \epsilon\text{Prob}[\Omega_1^\epsilon] = \epsilon e^{-\lambda\epsilon} \end{aligned} \geq \epsilon e^{-\lambda\epsilon}$$

It is then clear that (ii) holds for example for $\delta = \frac{1}{2}\epsilon e^{-\lambda\epsilon}$.

To prove (iii), we let $\Omega = \Omega_3 \cup \Omega_4$ where

$$\Omega_3 := \{\omega \in \Omega | \theta_\alpha(\omega) = \bar{h}\}, \quad \Omega_4 := \{\omega \in \Omega | \theta_\alpha(\omega) < \bar{h}\}, \tag{4.34}$$

for $\alpha = \bar{h}$ and note that

$$\begin{aligned} L(\alpha)_{\alpha=\bar{h}} &= \mathbb{E}[\theta_\alpha(\omega)] = \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_3}] + \mathbb{E}[\theta_\alpha(\omega)\mathbf{1}_{\Omega_4}] \\ &= \bar{h}\text{Prob}[\Omega_3] + \mathbb{E}[\theta_\alpha(\omega)|\omega \in \mathbf{1}_{\Omega_4}]\text{Prob}[\Omega_4] < \bar{h} \end{aligned}$$

where we used the fact that $\text{Prob}[\Omega_4] > 0$. To see this latter fact, note that, for η_α as in (4.29) and $h \in (\underline{h}, \bar{h})$, $\mathbb{E}[\eta_\alpha(h)] = \frac{h}{\alpha}g(h)$ from which $\{\omega \in \Omega | \eta_\alpha(\theta_\alpha(\omega)) \geq \frac{h}{\alpha}g(h)\} \subseteq \Omega_4$ has non-zero probability.

To establish (iv) consider positive scalars α_1, α_2 such that $\alpha_1 < \alpha_2$. Then for any $\omega \in \Omega$, we have

$$\theta_{\alpha_1}(\omega) \leq \theta_{\alpha_2}(\omega) \tag{4.35}$$

since $\eta_{\alpha_1}(\beta) > \eta_{\alpha_2}(\beta)$ for any $\beta > 0$, and therefore

$$\begin{aligned} \theta_{\alpha_1}(\omega) &= \min\left\{\beta \in [\underline{h}, \bar{h}] \;\middle|\; \eta_{\alpha_1}(\beta) \geq f(\beta_k)\right\} \\ &\leq \min\left\{\beta \in [\underline{h}, \bar{h}] \;\middle|\; \eta_{\alpha_2}(\beta) \geq f(\beta_k)\right\} = \theta_{\alpha_2}(\omega) \end{aligned}$$

From (4.35), we conclude $L(\alpha_1) = \mathbb{E}[\theta_{\alpha_1}] \leq \mathbb{E}[\theta_{\alpha_2}] = L(\alpha_2)$, that is, $L(\alpha)$ is non-decreasing for $\alpha > 0$. □

Since $L(\underline{h}) > \underline{h}$, $L(\bar{h}) < \bar{h}$, for $\underline{h} > 0$ and for $\underline{h} < \infty$, and L is non-increasing, we can simply plot $L(\alpha)$ for a dense grid of α and check when $\alpha \leq L(\alpha)$ (see Figure 4.4 for a numerical example). This approach can also be followed when $\bar{h} = \infty$ (by picking a sufficiently large $\bar{h}$) and, according to Lemma 4.2(ii), when $\underline{h} = 0$ (by picking a sufficiently small $\underline{h}$). Note that, in order to compute $\mathbb{E}[\theta_\alpha(\omega)]$, it suffices to consider the process (4.1), (4.2) with the event-triggered policy defined by (4.14), (4.30) in the interval $[0, \theta_\alpha]$. This can be achieved by running such a process in this interval and performing Monte Carlo simulations.

The next theorem is the main result of this chapter and states that this policy with the choice of α that satisfies (4.31) meets the first consistent property when f is concave (in particular, in the special cases of Figure 4.3) and also the second consistency property.

Theorem 4.3. Suppose that f is a concave function such that (4.25) holds for a given interval $(\underline{h}, \bar{h})$ and that either $\bar{h} < \infty$ or $\mathbb{E}[\theta_\alpha(\omega)] < \infty$. Let J_π be the performance of the proposed ETC policy for α such that (4.31) holds and let $\xi := \frac{\alpha}{L(\alpha)} \leq 1$. Then, if $\underline{h} = 0$,

$$J_\pi \leq \xi f(\mathbb{E}[\theta_\alpha(\omega)]) + \text{tr}(P\lambda W), \tag{4.36}$$

and if $f(h) = Ch$, $\underline{h} = 0$ and $\bar{h} = \infty$, then

$$J_\pi = \xi C\mathbb{E}[\theta_\alpha(\omega)] + \mathrm{tr}(P\lambda W). \tag{4.37}$$

Therefore, in such cases,

$$J_\pi < g(\mathbb{E}[\theta_\alpha(\omega)]) + \mathrm{tr}(P\lambda W).$$

Moreover, if $\bar{h} = \infty$, we have that $t_{k+1} > \min\{s_\ell | s_\ell > t_k.\ell \in \mathbb{N}_0\}$. Thus, in such a case, this is a consistent policy. Moreover, if $\bar{h} < \infty$ this is a $\bar{h}$-consistent policy.

□

Proof. We start by noticing that although the error (4.21) is in general discontinuous, the function $\eta_\alpha(t)$ is obtained by integrating a weighted norm of this error and therefore it is always continuous for $t \in [t_k, t_k + t)$, $t < \tau_k$. Thus, whenever there is a transmission we have the *equality*

$$\eta_\alpha(\tau_k) = f(\tau_k) \tag{4.38}$$

Moreover, due to our choice of α such that $\xi = \frac{\alpha}{\mathbb{E}[\theta_\alpha]} \leq 1$ we have that, for every $k \in \mathbb{N}_0$,

$$\xi\mathbb{E}[\eta_\alpha(\tau_k)] = \xi\mathbb{E}[\eta_\alpha(\theta_\alpha)] = g_{\mathrm{ETC}} \tag{4.39}$$

Then, if we take expected values on both side of (4.38) and multiply by ξ we obtain

$$g_{\mathrm{ETC}} = \xi\mathbb{E}[f(\theta_\alpha)]. \tag{4.40}$$

If $f(h) = Ch$, $\underline{h} = 0$ and $\bar{h} = \infty$ (assuming $\mathbb{E}[\theta_\alpha] < \infty$) we conclude that $g_{\mathrm{ETC}} = C\mathbb{E}[\tau]$ and from Theorem 4.2 we conclude (4.37).

If we still assume that $\underline{h} = 0$ and $\bar{h} = \infty$ (assuming $\mathbb{E}[\theta_\alpha] < \infty$) and let f be a concave function, we have from Jensen's inequality that

$$\mathbb{E}[f(\theta_\alpha)] \leq f(E[\theta_\alpha]).$$

Then, taking expected values on both side of (4.38) and using this inequality we conclude $g_{\mathrm{ETC}} \leq f(E[\theta_\alpha])$ and from Theorem 4.2 we conclude (4.36).

Consider now that $\underline{h} = 0$ but $\bar{h} < \infty$. We start by partitioning the probability space (4.32) into two sets Ω_3 and Ω_4 as in (4.34) but now for α such that $\alpha \leq \mathbb{E}[\theta_\alpha]$. For any realization of disturbances $\omega \in \Omega_4$ we have that (4.38) holds, whereas for $\omega \in \Omega_3$ we have that

$$\eta_\alpha(\bar{h}) \leq f(\bar{h}). \tag{4.41}$$

Then, if we let $c_1 := \text{Prob}[\Omega_3]$, $c_2 := \text{Prob}[\Omega_4]$,

$$\begin{aligned}\frac{g_{\text{ETC}}}{\xi} = \mathbb{E}[\eta_\alpha(\theta_\alpha)] &= \mathbb{E}[\eta_\alpha(\theta_\alpha)|\omega \in \Omega_3]c_1 + \mathbb{E}[\eta_\alpha(\theta_\alpha)|\omega \in \Omega_4]c_2 \\ &\leq f(\bar{h})c_1 + \mathbb{E}[f(\theta_\alpha)|\omega \in \Omega_4]c_2 \\ &\leq f(\bar{h})c_1 + f(\mathbb{E}[\theta_\alpha|\omega \in \Omega_4])c_2 \\ &\leq f(\bar{h}c_1 + \mathbb{E}[\theta_\alpha|\omega \in \Omega_4]c_2) \\ &= f(\mathbb{E}[\theta_\alpha])\end{aligned}$$

where $c_1 + c_2 = 1$, in the first inequality we used (4.41), in the second and third inequalities we used Jensen's inequality, and $\mathbb{E}[\theta_\alpha|\omega \in \Omega_3] < \bar{h}$, and in the last equality we used the fact that

$$\begin{aligned}\mathbb{E}[\theta_\alpha] &= \mathbb{E}[\theta_\alpha|\omega \in \Omega_3]c_1 + \mathbb{E}[\theta_\alpha|\omega \in \Omega_4]c_2 \\ &= \bar{h}c_1 + \mathbb{E}[\theta_\alpha|\omega \in \Omega_4]c_2\end{aligned}$$

Thus, from Theorem 4.2 we conclude (4.36).

The last part of the theorem follows by the definition of f and the definition of consistency. □

Note that, albeit transmissions in the loop are triggered with respect to a state-dependent mechanism making the closed-loop non-linear, when (4.37) holds, we obtain an affine relation between average cost and transmission rate. Moreover, note that due to Lemma 4.2, provided that $KW \neq 0$, we can always pick a concave f as in (4.27) and as represented in Figure 4.3.b with $\bar{h} < \infty$ (such that $\mathbb{E}[\theta_\alpha(\omega)] < \infty$). Therefore, we can always have a $\bar{h}$-consistent policy for any linear system for arbitrarily large $\bar{h}$.

Remark 4.1. (transmission rate) The proposed method is completely characterized by the choice of the function f and its domain $(\underline{h}, \bar{h})$. Clearly, if $f_1 \leq f_2$, then the average inter-transmission time and cost are larger for the method corresponding to f_2. For example, when $\underline{h} = 0$ and (4.26) holds, we can increase (decrease) the slope C to increase (decrease) the average inter-transmission time and cost. This will be illustrated in the simulation section (see Fig 4.5). However, since C is upper-bounded for the choice (4.37), the average inter-transmission time is also upper-bounded. To achieve a larger average inter-transmission time, we can select a larger $\underline{h}$ and different functions f.

Remark 4.2. (connection with dynamic ETC) Some recent works have proposed dynamic ETC (see, e.g., [37,45]) by which current transmission decisions depend on previous error state variables. In particular, the ETC has dynamic variables that filter error variables and the triggering policy depends on these dynamic variables. Note that our proposed event-triggered policy fits this description of dynamic ETC.

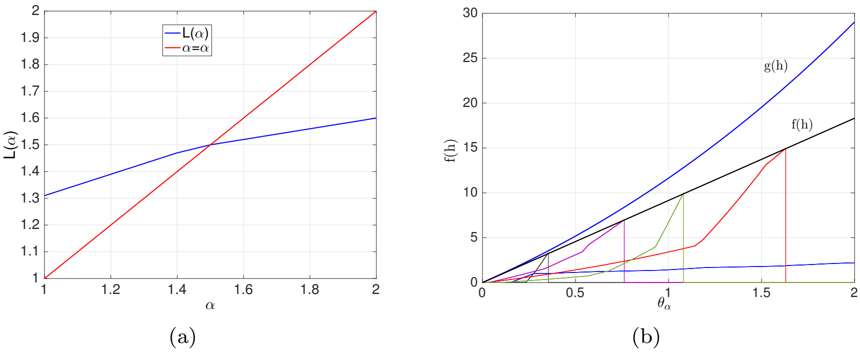


Figure 4.4. computation of α; any $\alpha \leq L(\alpha)$ is valid, we pick $\alpha = L(\alpha)$; (b) Illustration of the determination of the transmission times $\tau_k : \eta_\alpha(\tau_k) \geq f(\tau_k)$ for five sample paths

4.5 Examples

4.5.1 Performance of proposed ETC

We consider the ETC of a double integrator controlled as depicted in Fig 4.1. The process and cost parameters are given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad Q = I, \quad R = 10.$$

This leads to $K = \begin{bmatrix} -0.3162 & -0.8558 \end{bmatrix}$. For the disturbances/events, we take $\lambda = 5$ and consider $\{w_\ell\}_{\ell \in \mathbb{N}}$ to have normal distribution with covariance $W = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$, which influences the system through (4.2). Considering these numerical values, $g(h)$, described by (4.12), can be computed analytically. This results in

$$g(h) = 0.2083h^3 + 2.2553h^2 + 9.1557h,$$

which is a convex function for $h \geq 0$ and the right derivative of g at 0 is 9.1557. Therefore, $f(h) = Ch \quad$ for $C \in (0, \; 9.1557]$ satisfies (4.25). We take $\underline{h} = 0$ and $\bar{h} = 2$.

Considering $C = 9.1557$, we plot $L(\alpha)$ in Figure 4.4 and see that for $\alpha = 1.50$ we have $\alpha = \mathbb{E}[\theta_\alpha] = 1.5$. For this value of α, we obtain $g_{\text{ETC}} = 8.6393$ and the average cost $J = 30.0352$, since $\operatorname{tr}(P\lambda W) = 21.3959$. Figure 4.4(b) shows the result of applying the event-triggered mechanism (4.28) for 5 event sequences (Monte Carlo runs). Note that we can adjust $C \in (0, \; 9.1557)$ to trade average inter-transmission time $\mathbb{E}[\theta_\alpha]$ for average cost J_π. For each parameter C, we need to find a new value of $\alpha = \mathbb{E}[\theta_\alpha]$. Figure 4.5 shows how the inter-transmission

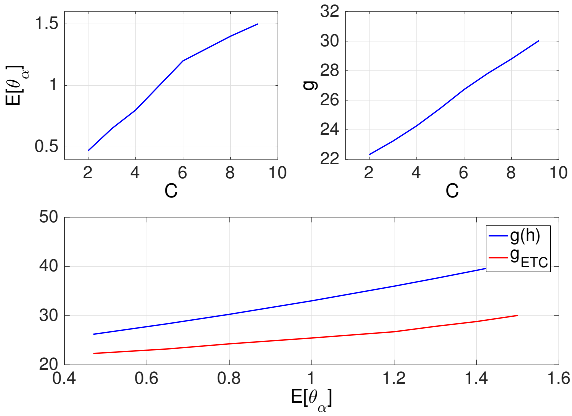


Figure 4.5. Top: average inter-transmission times (left) and performance as a function of the parameter C of the proposed ETC method. Bottom: Trade-off curves for periodic control and for the proposed ETC (computed for several values of C taken from the plots on top)

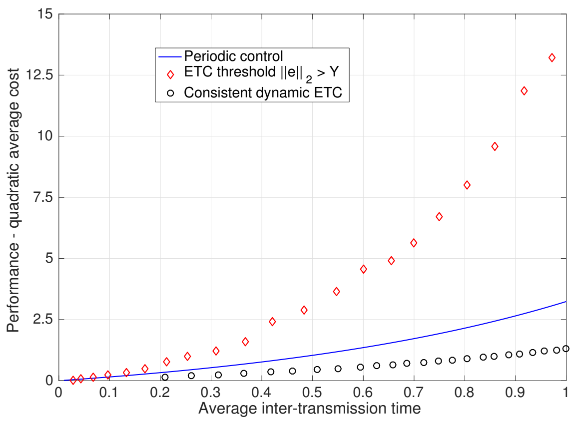


Figure 4.6. Trade-off curve average inter-transmission time version performance for three different policies

time and the performance changes with respect to this parameter C and plots the corresponding trade-off curve. From Figure 4.5 it is clear that the proposed method has the following consistency property: it achieves a better trade-off between average inter-sampling time and performance than periodic control.

4.5.2 Threshold-based policies are not necessary consistent

Consider the following process and cost parameters

$$A = \begin{bmatrix} 1 & \epsilon_1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 10 & 0 \\ 0 & \epsilon_2 \end{bmatrix}, R = \begin{bmatrix} 10 & 0 \\ 0 & \epsilon_2 \end{bmatrix}.$$

$\epsilon = 0.01$, $\epsilon_2 = 0.001$ and for the disturbances/events parameters $\lambda = 50$, $W = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.4 \end{bmatrix}$ with $\{w_\ell\}_{\ell \in \mathbb{N}}$ normally distributed with zero mean. We propose to compare three policies: (i) periodic control for which the cost $g(h)$, described by (4.12), can be computed symbolically and one can evaluate $\frac{d}{dh} g(h)_{|h=0} = 1.4274$; (ii) the proposed dynamic event-triggered policy for which we consider $\alpha = 0.2$, $\underline{h} = 0$, $\bar{h} = 10$, and different values of C such that $C\alpha < 1.4274$ so that (4.25) is met; (iii) a threshold-based ETC policy

$$t_{k+1} = t_k + \inf\{a_k \in [0, 10] \ \mid \ \|e(t_k + a_k)\|_2 \geq Y\}$$

where $\|e\|_2 := \sqrt{\sum_{i=1}^n e_i^2}$ is the Euclidean norm. We pick the following values for the following parameters $C \in \mathcal{C}$, $\mathcal{C} := \{0.25, 0.5, 0.75, \ldots, 6\}$, $Y \in \mathcal{C} \cup \{6.25, 6.5\}$ such that the average inter-transmission time belongs to the interval $[0, 1]$. The results are presented in Figure 4.6, showing the trade-off curves between average inter-transmission time and quadratic average cost performance for the three different policies. Note that the threshold-based policy, typically considered in the literature, is clearly not consistent since its performance is worse than that of periodic control for the same average inter-transmission time. In turn, the proposed policy satisfies this first consistency property, highlighting the advantages of designing ETC policies that are consistent by construction.

4.6 Conclusions

In this chapter, we defined two consistency properties for an ETC policy and we proposed a policy that meets these properties. While it is not hard to find an example where the second property of consistency does not hold, the effectiveness of ETC policies make it nontrivial to find an example when ETC policies perform worse than periodic control. However, we managed to provide an example of a linear quadratic control problem for which a traditional ETC policy, where transmissions occur if the Euclidean norm of the error between the system's state and a state estimate exceeds a threshold, did not satisfy the first consistency property. In Chapter 5, we will propose a consistent threshold-based policy using a different norm.

Chapter 5

A Consistent Threshold-based Policy for Event-triggered Control

In Chapter 4, we introduced the consistency properties for ETC and proposed a dynamic ETC policy that is consistent. Moreover, we illustrated in an example that threshold-based policies are not necessarily consistent. In this chapter[1] we propose a consistent threshold-based policy for periodic ETC in which transmissions are triggered if a special norm (different from the Euclidean) exceeds a certain threshold. This policy benefits from simplicity in implementation and builds upon a key result establishing the convexity of the trade-off curve between average cost performance vs. transmission rate for periodic control. Simulation results validate the strength of the proposed method.

5.1 Introduction

For many years, periodic control has been prevalent in digital control systems, due to its simple implementation and the existence of powerful techniques to analyze such systems. However, in the context of networked control systems (NCSs), periodic sampling and control lacks the flexibility one needs to efficiently manage the computation, communication, energy resources. This led to the advent of event-triggered control (ETC). The main idea behind ETC is to include state or output information for the determination of the times at which data-transmissions take place in a control loop. As such, in a networked control setting, the event-triggered controller aims at creating a balance between the

[1]This chapter is based on [68].

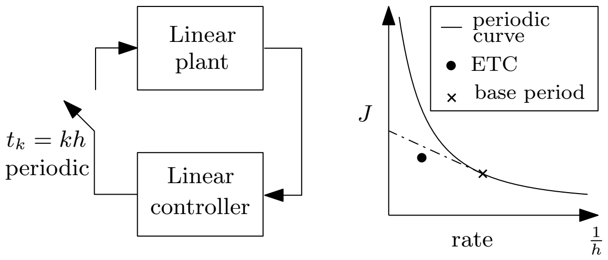
performance of the system and the use of feedback (in terms of the average transmission rate).

Extensive research has been conducted on ETC over the past decade. Several works (e.g. [33, 38, 41, 76, 79]) have generalized the idea in [28] by proposing to trigger transmissions from sensors (with computational capacities) to a remote controller when the norm of the error between the current state and a state estimate (available both at the sensor's computational unit and at the controller) exceeds a certain (possibly state or output-dependent) threshold. Indeed, various triggering mechanisms have been proposed in the literature including relative triggering [33, 76], mixed triggering [66, 76] and, recently, dynamic triggering [37, 45, 65]. Furthermore, different criteria have been analyzed for ETC systems, including stability [33], $\mathcal{L}_p$-gain performance [36, 37, 76] and quadratic cost criteria [38, 49, 59, 70]. When analyzing the performance based on a quadratic cost criteria, most of the works [39, 49, 54, 59, 63] formulate the event-triggered control problems in the framework of dynamic programming, and search for control policies that optimize the performance indices weighting both the state variables and the transmission rates in some way. Despite their analytical importance, the obtained dynamic programming formulations in the aforementioned works suffer from the curse of dimensionality and, therefore, most of the optimal triggering policies proposed in the literature are hard to implement in practice, and lack the insight and simplicity of the basic policies described in the pioneering works [30, 32, 33, 77].

Prompted by these observations, in some works suboptimal ETC with guarantees on the closed-loop performance and/or on the network usage have been proposed [40, 46, 48, 53, 59, 60, 90]. One can trace back the suboptimal ETC approach to the early work of [28], where it was shown that a threshold-based sampling and control outperforms the periodic control in terms of the variance of the state at the same sampling rate for a scalar linear system.

Motivated by these developments, the concept of consistency was introduced in Chapter 4 and a consistent ETC policy taking the form of dynamic ETC was proposed. To recall, an ETC policy is consistent if it possesses the following two properties: (i) It outperforms the performance of periodic control for the same average transmission rate; (ii) It requires no sensor updates (i.e., operates in open loop) in the absence of disturbances. Note that, the second consistency property typically only holds for ETC strategies in which a control-input-generator (CIG) based on system model is implemented in the actuator side and the full state (not the output) of the system is transmitted via the network, see, e.g., [41]. In fact, in a control loop, where a zero-order hold (ZOH) is used as the CIG, the second property can not be guaranteed in general for an ETC.

In Chapter 4 an example was provided in which a policy, where transmissions are triggered if the Euclidean norm of the error is larger than a threshold, performs worse (in a quadratic cost sense) than a periodic control policy at the same (average) transmission rate. This revealed that ad-hoc error threshold



$$J = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}[\int_0^T (x'Qx + u'Ru)dt]$$

Figure 5.1. Illustration of the key property to establish consistency in this chapter. For periodic control, the trade-off curve between average quadratic performance and transmission rate is convex.

policies may not necessary result in a better trade-off between performance and transmission rate than periodic control for systems with higher order than 1 (as considered in [28]). Or stated differently, threshold-based policies are not necessarily consistent.

As a consequence, in this chapter we are interested in the problem of designing a threshold policy, which is guaranteed to be consistent. Performance is measured by an average quadratic cost, as in the standard linear quadratic Gaussian (LQG) framework. While we consider continuous-time systems, the plant is only monitored periodically, at a fast rate, at which the transmission triggering condition is checked. Therefore, this can be seen as a periodic ETC (PETC) policy, see [76], where this term was introduced. The proposed solution builds upon a key result establishing the convexity of the trade-off curve between average quadratic cost performance and transmission rate for periodic control. In fact, we show that for the proposed ETC policy the pair (rate, performance) is below the tangent line of the periodic curve at the point corresponding to the base period of PETC policy. This property is illustrated in Figure 5.1. The resulting threshold-based policy will use a weighted norm generally different from the Euclidean norm. Compared to the proposed policy in Chapter 4, the new policy is simple to implement, because it does not require an integrator to realize the policy, and because the triggering condition is only evaluated at fixed periodic sampling times making it suitable for digital implementation. Indeed, since the triggering condition is only monitored at fixed-periodic sampling, a desired lower bound on minimum inter-event time can easily be tuned.

The remainder of the chapter is organized as follows. Section 5.2 formulates the problem and Section 5.3 gives the cost of periodic control. Section 5.4 provides the main result. Section 5.5 presents simulation results for a numerical example. Finally, in Section 5.6 the conclusions of the chapter are summarized.

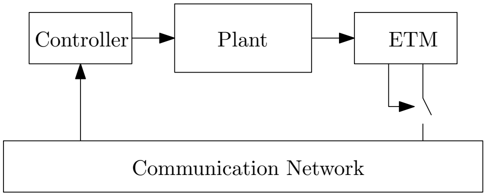


Figure 5.2. Considered control structure. The ETC unit closes the loop when triggering conditions, checked in the event-triggering mechanism (ETM), are met. The controller computes the control actions based on the received data or the predictions.

5.2 Problem Formulation

Figure 5.2 shows the control structure considered in this chapter. The plant model coincides with the jump linear model in Chapter 4 and is given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) & t &\in \mathbb{R}_{\geq 0} \backslash \{s_\ell\}_{\ell \in \mathbb{N}_0}, \\ x(s_\ell) &= x(s_\ell^-) + \omega_\ell, & l &\in \mathbb{N}_0, \end{aligned} \tag{5.1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state and $u(t) \in \mathbb{R}^{n_u}$ is the control input at time $t \in \mathbb{R}_{\geq 0}$, and $\{\omega_\ell\}_{\ell \in \mathbb{N}_0}$, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, is an independent and identically distributed (i.i.d.) sequence of random variables with zero mean and covariance R_w. Moreover, $\{s_\ell\}_{\ell \in \mathbb{N}_0}$ denotes the sequence of times at which the disturbance impacts the state, where $s_0 = 0$ and $s_{\ell+1} - s_\ell$, $\ell \in \mathbb{N}_0$, are assumed to be independent and exponentially distributed with rate λ, i.e.,

$$\text{Prob}[s_{\ell+1} - s_\ell > a] = e^{-\lambda a} \quad \ell \in \mathbb{N}_0, \tag{5.2}$$

and $x(s_\ell^-) := \lim_{\epsilon \to 0, \epsilon > 0} x(s_\ell - \epsilon)$. The event-triggering mechanism (ETM) samples the state of the plant at $t_k \in \mathbb{R}_{\geq 0}$ for $k \in \mathbb{N}_0$, and decides whether or not to transmit the value of the plant state $x(t_k)$ to the controller. Here, $k \in \mathbb{N}_0$ is the sample counter. Note that this model can capture the behavior of the more common model used in literature where the process disturbances are modeled by a Wiener process (see [88]). In order to model the decision making process for the transmissions, we introduce $\sigma_k \in \{0,\ 1\} =: \mathcal{M}$, where $\sigma_k = 1$ means that at sample time t_k a transmission takes place and $\sigma_k = 0$ indicates that no transmission takes place at time t_k, $k \in \mathbb{N}_0$. At both the controller and the ETM, see Figure 5.2, there is a model-based control-input generator (CIG), which uses the linear estimator

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) & t &\in \mathbb{R}_{\geq 0} \backslash \{t_k \mid \sigma_k = 1,\ k \in \mathbb{N}_0\}, \\ \hat{x}(t) &= x(t) & t &\in \{t_k \mid \sigma_k = 1,\ k \in \mathbb{N}_0\}, \end{aligned} \tag{5.3}$$

with initial condition $\hat{x}_0 = x_0$, to generate the state estimate $\hat{x}(t)$ at time $t \in \mathbb{R}_{\geq 0}$. Moreover, the controller unit produces the control input $u(t)$, $t \in \mathbb{R}_{\geq 0}$, fed into the plant. Note that, as will be clear in the sequel, the control input $u(t)$ is also known by the ETM.

In this chapter, we assume that the sampling times $t_k,\ k \in \mathbb{N}_0$, are equidistant, i.e., $t_k = k\tau$, where τ is a positive constant. Therefore, transmissions can only be triggered at integer multiples of the sampling period τ. In other words, if $\kappa_m \in \mathbb{R}_{\geq 0}$ denotes the m-th transmission time, where $m \in \mathbb{N}_0$ is a counter for the transmissions, then we have that

$$\{\kappa_m \mid m \in \mathbb{N}_0\} = \{k\tau \mid k \in N_0,\ \sigma_k = 1\}, \tag{5.4}$$

which is the set of transmission times as already used in (5.3). As already mentioned in the introduction, this class of ETC policies is known as periodic ETC [76].

We denote by $I^c(t)$ the available information at the controller at time $t \in R_{\geq 0}$, which is given by

$$I^c_m(t) = \{x(\kappa_m) \mid m \in \mathbb{N}_0, \kappa_m \leq t\}. \tag{5.5}$$

Furthermore, by I^s_k we denote the available information at the scheduler (ETM) at time $t_k, k \in \mathbb{N}_0$, given by

$$I^s_k = (I^s_{k-1}, x_k, \hat{x}_k) \quad \text{for } k \in \mathbb{N}_{\geq 1}, \tag{5.6}$$

where $x_k := x(t_k^-),\ \hat{x}_k := \hat{x}(t_k^-)$ and $I^s_0 = x_0$.

A control policy γ is a set of functions ν_m that map the available information at the controller at time t, i.e., $I^c_m(t)$, to the control actions $u(t)$, and hence, $u(t) = \nu_m(t,\ I^c_m(t))$. Similarly, the transmission policy θ is a set of functions $\mu_k : I^s_k \mapsto \mathcal{M}$ that map the available information at the ETM at time t_k, i.e., I^s_k, to the transmission decision set $\mathcal{M},\ k \in \mathbb{N}_0$. Finally, a policy π consists of a control policy and a transmission policy, i.e., $\pi := (\gamma, \theta)$.

In this chapter, we are interested in designing control and transmission policies that outperform periodic time-triggered policies in terms of the trade-off between the performance criterion, V, and the average transmission, r, defined as

$$V := \limsup_{T\to\infty} \frac{1}{T}\,\mathbb{E}[\int_0^T x(t)^\intercal Q x(t) + u(t)^\intercal R u(t) dt] \tag{5.7}$$

$$r := \frac{1}{\tau} \times \limsup_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} \mathbb{E}[\sigma_k], \tag{5.8}$$

where Q and R are positive-definite matrices. Moreover, we require that the policy is designed such that no new transmissions are triggered (i.e., the system operates in open loop) in the absence of disturbances. These requirements were formally defined as the consistency properties in the context of ETC in Chapter 4 and are restated next, in which we use the notation $V_p(r)$ to denote the optimal cost (5.7) of the periodic control policy p with the transmission rate $r > 0$ that minimizes (5.7). This optimal periodic policy and corresponding cost will be computed in the next section.

Definition 4.1. Let V_π and r_π denote the cost (5.7) and the average transmission rate (5.8) of a policy π, respectively. We call the policy π is consistent if:

1. $V_\pi \leq V_p(r_\pi)$, i.e., π achieves a better performance than that of the optimal periodic control policy for the same average transmission rate r_π.

2. Along any closed-loop trajectory given by (5.1)-(5.3) and policy π (and thus some realization given by $\{s_l\}_{l\in\mathbb{N}_0}$ and $\{w_l\}_{l\in\mathbb{N}_0}$ of the disturbance) it holds that $\kappa_{m+1} \geq \bar{s}_m$ for every $m \in \mathbb{N}_0$, where

$$\bar{s}_m := \min\{s_\ell \mid s_\ell > \kappa_m, \ell \in \mathbb{N}_0\},$$

   i.e., the scheduler generates no new transmissions after a given transmission time $\kappa_m,\ m \in \mathbb{N}_0$, as long as no disturbances act on the plant.

With this definition, the problem formulation can be stated as: *Design a (easy-to-implement) consistent control and transmission policy for the system configuration discussed above.*

5.3 Periodic all-time transmission policy

Before proposing our novel consistent PETC strategies, in this section we first present the optimal *periodic* policies and the corresponding performance and a few preliminary but instrumental observations leading to our main results. The optimal periodic policy p corresponds to the constant map $\mu_k(I_k^s) = 1,\ k \in \mathbb{N}_0$, for the transmission policy, i.e., $\kappa_m = t_m,\ m \in \mathbb{N}_0$, in other words that the transmission times are equal to the sampling times, while applying the control policy

$$u(t) = L\hat{x}(t), \quad t \in \mathbb{R}_{\geq 0} \tag{5.9}$$

with

$$L = -R^{-1}B^\intercal P, \tag{5.10}$$

where P is the unique positive-definite solution of the continuous-time algebraic Ricatti equation (CARE)

$$A^\intercal P + PA - PBR^{-1}B^\intercal P + Q = 0. \tag{5.11}$$

This policy results in the average cost

$$V_p(r_p) = \lambda \operatorname{Tr}(PR_\omega) + r_p \int_0^{\frac{1}{r_p}} \operatorname{Tr}(L^\intercal RL\Sigma(t))dt, \tag{5.12}$$

where $r_p := \frac{1}{\tau}$ is the transmission rate and

$$\Sigma(s) = \lambda \int_0^s e^{Ar} R_w e^{A^\intercal r} dr \tag{5.13}$$

corresponds to the covariance of the error

$$e(t) := x(t) - \hat{x}(t) \tag{5.14}$$

at time $t = t_k + s$, i.e., $\Sigma(s) = \mathbb{E}[e(t_k + s)e^\intercal(t_k + s)]$ for any $k \in \mathbb{N}_0$ (c.f. [67]).

Before introducing our proposed PETC policy in the next section, we state the following key result upon which we build the PETC.

Theorem 5.1. The performance index (5.7) of the periodic policy p with respect to the transmission rate r_p is convex, i.e., the mapping $V_p : [0, \infty) \to [0, \infty)$ given by $r_p \mapsto V_p(r_p)$ is a convex function. □

The proof of Theorem 5.1 follows from the following lemma.

Lemma 5.1. The function $g : \mathbb{R}_{>0} \to \mathbb{R}$, given by $g(r) = r \int_0^{\frac{1}{r}} f(s)ds$ is convex if f

- is continuously differentiable, and
- is non-decreasing, i.e., $\frac{d}{ds} f(s) \geq 0,\ s \geq 0$.

Proof of Lemma 5.1. Since f is continuously differentiable, g is twice continuously differentiable and

$$\frac{d^2 g}{dr^2}(r) = \frac{1}{r^3} \frac{df}{dr}(\frac{1}{r}),$$

which is non-negative for $r \in \mathbb{R}_{>0}$ (as f is monotonically increasing), therefore, g is convex. □

Proof of Theorem 5.1. The function $s \mapsto \mathrm{Tr}(L^\intercal R L \Sigma(s))$ satisfies both conditions of Lemma 5.1. Note that $L^\intercal R L$ and $\frac{d}{ds}\Sigma(s) = \lambda e^{As} R_w e^{A^\intercal s},\ s \in \mathbb{R}_{\geq 0}$, are positive semi-definite matrices, and hence,

$$\mathrm{Tr}(L^\intercal R L \frac{d}{ds}\Sigma(s)) \geq 0,\ s \in \mathbb{R}_{\geq 0}.$$

Therefore, the function $r_p \mapsto V_p(r_p)$ is convex. □

Theorem 5.1 provides the corner stone for introducing our proposed consistent PETC (CPETC). This originates from the fact that the graph of a convex function is above any tangent plane. Therefore, if we can provide a policy whose performance is bounded by the tangent of the optimal periodic policy, we ensure the first consistency property. This is illustrated in Figure 5.1. In next section, we present the main result of this chapter.

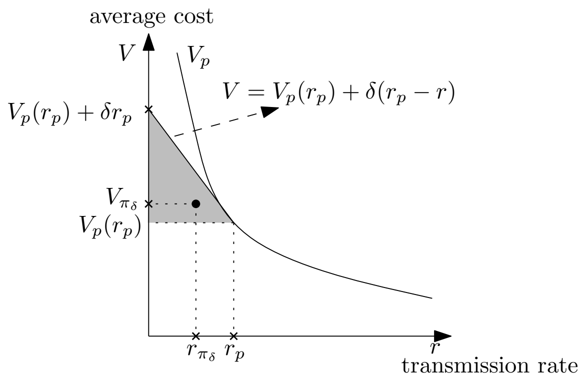


Figure 5.3. Illustration of the trade-off between average cost and transmission rate for the proposed triggering policy, where δ is chosen as in (5.17). The gray area represents the performance region for the proposed triggering algorithm characterized by (5.18)-(5.20).

5.4 Consistent Periodic ETC Policy

The following theorem formally introduces the proposed consistent PETC (CPETC).

Theorem 5.2. Let $r_p > 0$ be given. Consider system (5.1) with the control policy $u(t) = L\hat{x}(t),\ t \in \mathbb{R}_{\geq 0}$, as in (5.3) and (5.9). Then the ETC policy π_δ with the triggering policy θ_δ given by

$$\sigma_k = \mu_k(I_k^s) = \begin{cases} 1, & e_k^\intercal \Gamma(r_p) e_k > \delta \\ 0, & \text{otherwise,} \end{cases} \tag{5.15}$$

where $e_k := e(t_k^-)$, $t_k = k\frac{1}{r_p},\ k \in \mathbb{N}_0$, and

$$\Gamma(\tau) = \int_0^\tau e^{A^\intercal t} L^\intercal R L e^{At} dt \tag{5.16}$$

is consistent for any δ such that

$$0 \leq \delta \leq -\frac{d}{dr_p} V_p(r_p) = \mathrm{Tr}(L^\intercal R L(\frac{1}{r_p}\Sigma(1/r_p) - \int_0^{1/r_p} \Sigma(t)dt)). \tag{5.17}$$

In particular, for any $\delta \in \mathbb{R}_{\geq 0}$ we have

$$V_{\pi_\delta} + \delta r_{\pi_\delta} \leq V_p(r_p) + \delta r_p \tag{5.18}$$

$$r_{\pi_\delta} \leq r_p, \tag{5.19}$$

where V_{π_δ} is the cost (5.7) of policy π_δ and r_{π_δ} is the average transmission rate of policy π_δ as defined in (5.8). In addition, when $\delta \leq -\frac{d}{dr_p}V_p(r_p)$ as in (5.17), then

$$V_p(r_p) \leq V_{\pi_\delta} \leq V_p(r_{\pi_\delta}). \tag{5.20}$$

□

Figure 5.3, shows the cost and the average transmission rate trade-off for the proposed policy. The gray area represents the guaranteed performance region characterized by (5.18)-(5.20) for the parameter δ chosen as in (5.17). Note that δ appears as a tuning knob which influences the slope of the upper bound defined by the line $\{(V,r) \mid V = V_p(r_p) + \delta(r_p - r),\ r \in (0,\ r_p]\}$. Furthermore, the higher the value of δ, the less stringent the triggering condition (5.15) leading to a lower transmission rate and a larger performance region.

Proof of Theorem 5.2. To prove Theorem 5.2 we need some preliminaries. Consider the finite horizon performance index

$$\bar{V} := \frac{1}{T}\mathbb{E}[\int_0^T x(t)^\intercal Q x(t) + u(t)^\intercal R u(t) dt] \tag{5.21}$$

with $T = N\tau,\quad N \in \mathbb{N}$. Since the transmission decisions can take place at discrete times $t_k = k\tau$ for $k \in \mathcal{F} = \{0, 1, \dots, N-1\}$, we divide the integration interval $[0,\ T]$ of (5.21) into N equidistant subintervals of length τ. We assume that we apply the control policy $u = L\hat{x}$ as in (5.3) and (5.9). Based on similar arguments as provided in [67, eq.(37)], we then obtain

$$\begin{aligned}\mathbb{E}[\int_{t_k}^{t_k+\tau} & x(t)^\intercal Q x(t) + u(t)^\intercal R u(t) dt \mid x(t_k)] = \\ & x(t_k)^\intercal P x(t_k) + \tau\,\mathrm{Tr}(\lambda R_w) - \mathbb{E}[x(t_k+\tau)^\intercal P x(t_k+\tau) \mid x(t_k)] + \\ & \qquad \mathbb{E}[\int_{t_k}^{t_k+\tau} (u(t) - Lx(t))^\intercal R(u(t) - Lx(t)) dt \mid x(t_k)].\end{aligned} \tag{5.22}$$

Adding this equation from $k = 0$ until $k = N-1$ and dividing by $T = N\tau$, taking expected values and using the fact that $\mathbb{E}[. \mid x(t_k)] = \mathbb{E}[.]$, we obtain

$$\bar{V} = \frac{1}{N\tau}x(0)^\intercal P x(0) + C_e - \frac{1}{N\tau}\mathbb{E}[x(N\tau)^\intercal P x(N\tau)] + \mathrm{Tr}(P\lambda R_\omega), \tag{5.23}$$

where

$$C_e := \frac{1}{N\tau}\mathrm{Tr}(L^\intercal R L \sum_{i=0}^{N-1} \int_{i\tau}^{(i+1)\tau} \mathbb{E}[e(s)e(s)^\intercal \mid e(t_i)] ds). \tag{5.24}$$

Using [67, eq.(45)] with initial condition $e(t_k)$, we obtain

$$\begin{aligned}\mathbb{E}[e(t_k+r)e(t_k+r)^\intercal \mid e(t_k)] = e(t_k)e(t_k)^\intercal \\ + A\int_{t_k}^{t_k+r}\mathbb{E}[e(s)e(s)^\intercal \mid e(t_k)]ds \\ + \int_{t_k}^{t_k+r}\mathbb{E}[e(s)e(s)^\intercal \mid e(t_k)]dsA^\intercal + r\lambda R_\omega.\end{aligned} \tag{5.25}$$

Defining $W(r) := \mathbb{E}[e(t_k+r)e(t_k+r)^\intercal \mid e(t_k)]$, (5.24) can be rewritten as

$$W(r) = e(t_k)e(t_k)^\intercal + A\int_0^r W(s)ds + \int_0^r W(s)dsA^\intercal + r\lambda R_\omega, \tag{5.26}$$

which is a Volttera equation of second type with unique solution [91] given by

$$\begin{aligned}W(r) &= e^{Ar}e(t_k)e(t_k)^\intercal e^{A^\intercal r} + \Sigma(r)\\ \Sigma(r) &= \int_0^r e^{As}\lambda R_w e^{A^\intercal s}ds.\end{aligned}$$

Therefore, we can rewrite the term under summation in (5.24) as

$$\int_{i\tau}^{(i+1)\tau}\mathbb{E}[e(s)e(s)^\intercal \mid e(t_i)]ds = \int_0^\tau \Sigma(t)dt + \int_0^\tau e^{At}\,\mathbb{E}[e(i\tau)e(i\tau)^\intercal]e^{A^\intercal t}dt. \tag{5.27}$$

Consequently, replacing (5.27) in (5.24), we obtain

$$\sum_{i=0}^{N-1}\int_{i\tau}^{(i+1)\tau}\mathbb{E}[e(s)e(s)^\intercal]dt = \int_0^\tau e^{At}\sum_{i=0}^{N-1}\mathbb{E}[e(i\tau)e(i\tau)^\intercal]e^{A^\intercal t}dt + N\tau\int_0^\tau \Sigma(t)dt, \tag{5.28}$$

which leads to

$$C_e = \int_0^\tau \mathrm{Tr}(L^\intercal RL\Sigma(t))dt + \frac{1}{N\tau}\mathrm{Tr}(\Gamma(\tau)\sum_{i=0}^{N-1}\mathbb{E}[e(i\tau)e(i\tau)^\intercal]), \tag{5.29}$$

where $\Gamma(\tau)$ is as defined in (5.16). Due to the possibility of a transmission event at t_k for $k \in \mathcal{F}$, we can obtain $e(t_k) = 0$. This is key to the proposed policy, as the performance index (5.21) can be reformulated as

$$\begin{aligned}\bar{V} = \lambda \operatorname{Tr}(PR_\omega) + \int_0^\tau \operatorname{Tr}(L^\intercal RL\Sigma(t))dt \\ + \frac{1}{N\tau}\sum_{i=0}^{N-1}\mathbb{E}[e(t_i)^\intercal\Gamma(\tau)e(t_i)] - \frac{1}{N\tau}\mathbb{E}[x(T)^\intercal Px(T)] \\ + \frac{1}{N\tau}\mathbb{E}[x(0)^\intercal Px(0)]. \end{aligned} \tag{5.30}$$

We focus on the third term and introduce the function $\bar{J}^N$ with and additional additive constant $\delta \in \mathbb{R}_{\geq 0}$ as the cost of transmission at time t_k, i.e.,

$$\bar{J}^N := \frac{1}{N\tau}\sum_{k=0}^{N-1}\mathbb{E}[e_k^\intercal\Gamma(\tau)e_k + \delta\sigma_k], \tag{5.31}$$

where $e_k = e(k\tau)$. Note that for the periodic all-time transmission policy, we have $\sigma_k = 1$ and $e_k = 0$ for every $k \in \mathcal{F}$. Therefore,

$$\bar{J}_p^N(r_p) = \delta\frac{1}{\tau} = \delta r_p, \tag{5.32}$$

where $\bar{J}_p^N(r_p)$ is (5.31) for $\sigma_k = 1,\ k \in \mathcal{F}$. Applying the proposed policy (5.15) leads to

$$\bar{J}_{\pi_\delta}^N \leq \delta r_p = \bar{J}_p \quad \text{for every } N \in \mathbb{N}, \tag{5.33}$$

which is due the fact that the policy (5.15) imposes

$$e_k^\intercal\Gamma(\tau)e_k + \delta\sigma_k = \min\{e_k^\intercal\Gamma(\tau)e_k, \delta\} \leq \delta. \tag{5.34}$$

Since the inequality (5.33) holds for all $N \in \mathbb{N}$, it also holds for the limit, i.e.,

$$J_{\pi_\delta} := \lim_{N\to\infty}\bar{J}_{\pi_\delta}^N \leq \lim_{N\to\infty}\bar{J}_p^N =: J_p = \delta r_p \tag{5.35}$$

is well defined.

Applying the policy (5.15) and taking the limit as $T \to \infty$ (equivalently $N \to \infty$) from both sides of (5.30), on the left-hand side, we recover the performance index (5.7) for the policy π_δ. On the right-hand side, the summation term is bounded due to (5.35) and the last term converges to zero. Moreover, following a similar reasoning as in [67, eq. (47)] the fourth term vanishes as

$$\mathbb{E}[x(T)^\intercal x(T)] \leq C, \quad \text{for every } T \in \mathbb{R}_{\geq 0}, \tag{5.36}$$

which is equivalent to mean square stability of the system (5.1) applying the control policy (5.3) and triggering policy (5.15). Therefore,

$$V_{\pi_\delta} = \underbrace{\lambda \operatorname{Tr}(PR_\omega) + \frac{1}{\tau}\int_0^\tau \operatorname{Tr}(L^\intercal RL\Sigma(t))dt}_{V_p(r_p)} + \frac{1}{\tau}\lim_{N\to\infty}\frac{1}{N}\sum_{i=0}^{N-1}\mathbb{E}[e_i^\intercal\Gamma(\tau)e_i]. \tag{5.37}$$

It is interesting to note that the cost now is divided in three parts. The first term is the continuous-time optimal cost. The second term incurs due to periodic sampling and the third term is the contribution of the application of the event-triggered policy whereby in the special case of periodic transmissions, i.e., $\mu_k(I_k^s) = 1$ for $k \in \mathbb{N}_0$, the third term vanishes and we recover the cost of periodic all-time transmission as in (5.12) with transmission period τ. Furthermore, combining (5.35) and (5.37) leads to

$$V_{\pi_\delta} + \delta r_{\pi_\delta} \leq V_p(r_p) + \delta r_p. \tag{5.38}$$

Also note that by construction

$$r_{\pi_\delta} \leq r_p. \tag{5.39}$$

In order to prove the first consistency property of the proposed policy note that in $V-r$ coordinates $V(r) - V_p(r_p) = -\delta(r - r_p)$ characterizes a line passing through $(V_p(r_p),\ r_p)$ (see Figure 5.3). Since the curve of periodic performance vs rate is convex according to Theorem 5.1, we have

$$V_p(y) \geq V_p(x) + \frac{d}{dx}V_p(x)(y-x) \tag{5.40}$$

for any $x \in \mathbb{R}_{\geq 0}$ and for any $y \in \mathbb{R}_{\geq 0}$. If we set $x = r_p$ and $y = r_{\pi_\delta}$, we obtain

$$V_p(r_{\pi_\delta}) \geq V_p(r_p) + \frac{d}{dr_p}V(r_p)\underbrace{(r_{\pi_\delta} - r_p)}_{\geq 0}. \tag{5.41}$$

Choosing

$$\delta = -\frac{d}{dr_p}V_p(r_p) = Tr(L^\intercal RL(\frac{1}{r_p}\Sigma(1/r_p) - \int\limits_0^{1/r_p} \Sigma(t)dt)),$$

then from (5.38) and (5.41), we conclude

$$V_p(r_{\pi_\delta}) \geq V_p(r_p) - \delta(r_{\pi_\delta} - r_p) \geq V_{\pi_\delta}. \tag{5.42}$$

Moreover, due to (5.39), (5.42) holds also for any $0 \leq \delta \leq -\frac{d}{dr_p}V_p(r_p)$. Also note that due to (5.37) the obtained cost of π_δ is at least equal to the cost of periodic policy with rate r_p therefore

$$V_p(r_p) \leq V_{\pi_\delta}. \tag{5.43}$$

Note that in the case of no disturbance after a given transmission time κ_m we have $e(t) = 0,\ t > \kappa_m$, and, therefore, there is no transmission for $t > \kappa_m$ (the second consistency criterion holds). Moreover, due to (5.42) the proposed policy outperforms the periodic control with the same average transmission rate (the first consistency criterion holds). Hence, for any $\delta \in \mathbb{R}_{\geq 0}$ satisfying (5.17), policy π_δ is consistent. $\square$

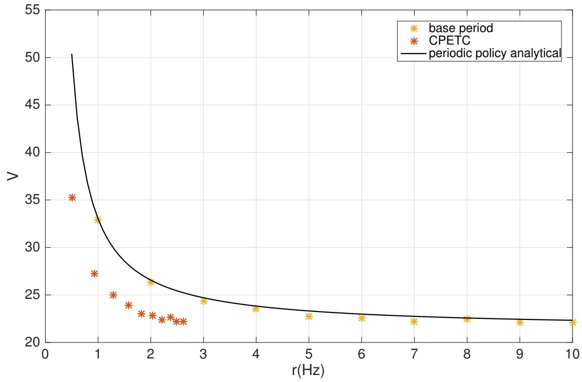


Figure 5.4. Performance of the proposed CPETC for control of a double integrator system over a communication network. The solid line represents the analytical performance (5.45). The yellow stars represent the periodic control used as base policies via Montecarlo simulations and the red stars illustrate the performance trade-off of the corresponding consistent periodic event-triggered policy.

5.5 Simulation results

In this section we illustrate the proposed ETC policy on a double integrator controlled over a communication network as depicted in Figure 5.2. The process and cost parameters are given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad Q = I; \quad R = 10. \tag{5.44}$$

This leads to $L = \begin{bmatrix} 0.3162 & 0.8558 \end{bmatrix}$ in (5.10). For the disturbances, we take $\lambda = 5$ and we consider that the w_ℓ, $\ell \in \mathbb{N}_0$, have a normal distribution with covariance $W = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$, which influences the system through (5.1). Considering these numerical values, V_p, described by (5.12), can be computed analytically as

$$V_p(r_p) = 21.3959 + 9.1557\frac{1}{r_p} + 2.2553\frac{1}{r_p^2} + 0.2083\frac{1}{r_p^3}. \tag{5.45}$$

Moreover, the derivative of V_p is

$$\frac{d}{dr_p}V_p(r_p) = -(0.6249\frac{1}{r_p^4} + 4.5106\frac{1}{r_p^3} + 9.1557\frac{1}{r_p^2}).$$

We consider the periodic policy with various transmission rates and the corresponding thresholds. Figure 5.4 illustrates the results of the simulation. The yellow stars illustrate the considered periodic policies with cost computed through

Monte-Carlo simulations. The black line corresponds to the analytical performance of the periodic policies as in (5.45) and the red stars correspond to the applied CPETC for various value of sampling rate r_p computed via Monte-Carlo simulations. It is clearly visible that the triggering condition not only creates a better cost vs transmission ratio trade-off but also significantly reduces the transmission rate.

5.6 Conclusions

In this chapter a *consistent* periodic ETC policy was introduced. The proposed policy is consistent in the sense that (i) it outperforms the performance of periodic control for the same average transmission rate and (ii) it does not trigger sensor updates in the absence of disturbances. We showed that consistency can always be achieved considering linear systems, state feedback, and average quadratic costs as the performance measurement using the key observation that the average quadratic cost for periodic policies is a convex function of the transmission rate. This resulted in an error-based threshold policy using the weighted norm of the error as in (5.15) instead of the Euclidean norm. The resulting policy is easy-to-implement due to its sampled-data nature and the simple threshold-based triggering mechanism.

Part III

Experimental validation and application to ground robotics

Chapter 6

Experimental validation of an event-triggered policy for remote sensing and control with performance guarantees

As the theory of ETC matures, there is a need to experimentally validate and test these methods in applications of interest. In this chapter[1], we extend and experimentally validate the ETC strategy presented in Chapter 3 for the remote point-stabilization problem of a ground robot. This strategy specifies when transmissions should occur in both sensor to controller and controller to actuator channels, and guarantees a bound on the finite-horizon quadratic cost. The experimental results are coherent with the simulation results and reveal that ETC can lead to a tremendous data transmission reduction (in the considered set-up up to 90%) with respect to periodic time-triggered control, with just a minor performance loss.

6.1 Introduction

Several ETC methods are currently available in the literature but not many provide experimental validations (see, e.g., [21,92–96]). Some of the early works [29, 30,33,74,76,97] proposed threshold policies in which transmissions are triggered if the error between previously sent and current state, output or control variables exceeds given thresholds, either constant or proportional to state, output

[1]This chapter is based on [64].

or control variables. Research on ETC is now being pursued in several directions, such as non-linear [36, 45, 98], optimal [38, 39] and suboptimal ETC [47, 48, 59]. However, while the ETC theory is maturing at a fast pace, there are limited experimental validations of the ETC methods proposed in the literature. Some experimental validations can be found in [92–96, 99] but apart from these and a few others in the literature, it is fair to state that the (experimental) application of ETC is in unbalance with the high number of theoretical contributions.

The purpose of the present chapter is to experimentally validate the ETC strategy proposed in Chapter 3. This strategy specifies when transmissions should occur in both sensor to controller and controller to actuator channels, and guarantees a bound on the performance measured by an infinite-horizon quadratic cost. In this chapter, we consider a performance index in terms of a finite-horizon quadratic cost. Following the developments in Chapter 3, we can establish that the corresponding policy for the finite-horizon cost provides a performance bound with respect to a periodic control scheme, where transmissions are triggered at the maximum allowable rate, while reducing the number of transmissions.

The experimental set-up consists of a robot, a wired camera, a control unit, and a wireless network. The camera is used to obtain images of the robot in its work space. The controller queries an image of the work space from the camera and processes this image to get the estimated position of the robot with respect to a certain reference frame. Based on the estimated position of the robot the controller is able to compute new control actions. These computed control actions are sent through a wireless network to the actuators of the robot at designated time instances, which are defined by the controller.

The experiments validate the usefulness of the ETC policy derived in Chapter 3 and are coherent with simulation results also presented in the present chapter. The benefits in terms of communication reduction are tremendous. In fact, the communication is reduced by 80% and 90% for the sensor and the actuator network, respectively, while guaranteeing the performance bounds on the cost with respect to the all-time transmission policy.

The remainder of the chapter is organized as follows. Section 6.2 formulates the problem. In Section 6.3, we state the control policy which will be validated in this chapter. In Section 6.4, the physical motion system on which the control policy is implemented is discussed. Section 6.5 presents numerical simulations of the ETC policy based on the model of the motion system. Section 6.6 presents experimental results of the ETC policy implemented on the actual motion system. In Section 6.7, the results obtained by numerical simulations are compared to the experimental results. Section 6.8 provides concluding remarks and directions for future work.

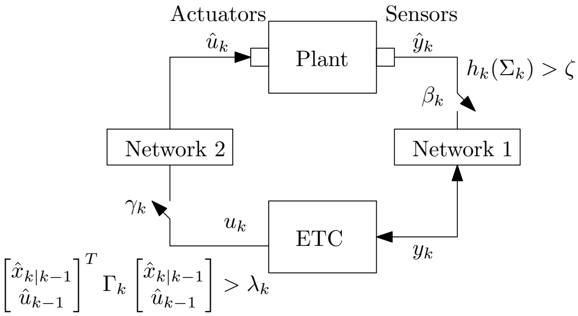


Figure 6.1. Overall set-up and proposed policy in which the sensor query depends only on the Kalman filter covariance matrices Σ_k, while the control update transmissions are scheduled based on the Kalman filter state estimate $\hat{x}_{k|k-1}$, and the previously sent control input $\hat{u}_{k-1}$.

Nomenclature

The trace of a square matrix $A \in \mathbb{R}^{n \times n}$ is denoted by $\mathrm{Tr}(A)$. The expected value vector and the covariance matrix of a random variable $\eta \in \mathbb{R}^n$ are denoted by $\mathbb{E}[\eta]$ and $\mathrm{Cov}[\eta]$, respectively. For a symmetric matrix $Z \in \mathbb{R}^{n \times n}$, we write $Z \succ 0$ ($Z \succeq 0$) to denote that Z is positive definite (positive semi-definite). The identity map is denoted by id and $\circ$ denotes composition operator. The finite set $\mathbb{S} = \{0, \ldots, N-1\}$ is defined for $N \in \mathbb{N}$ (assuming N is clear from the context).

6.2 Problem formulation

Figure 6.1 shows the block diagram of the considered networked-control system. We assume that the plant operates as a linear discrete-time system described by

$$\begin{aligned} x_{k+1} &= Ax_k + B\hat{u}_k + v_k \\ \hat{y}_k &= Cx_k + r_k, \end{aligned} \tag{6.1}$$

where $x_k \in \mathbb{R}^{n_x}$ for $k \in \mathbb{S} \cup N$, denotes the state, and $\hat{u}_k \in \mathbb{R}^{n_u}$ and $\hat{y}_k \in \mathbb{R}^{n_y}$ for $k \in \mathbb{S}$, denote the input and the output, respectively. Furthermore, v_k and r_k represent the state disturbance and measurement noise at time $k \in \mathbb{S}$. We assume that the disturbance $\{v_k\}_{k \in \mathbb{S}}$ and noise $\{r_k\}_{k \in \mathbb{S}}$ processes consist of sequences of independent zero-mean Gaussian random vectors with covariances Φ_v and Φ_r, respectively. The initial state is assumed to be either a Gaussian random variable with mean $\hat{x}_0$ and covariance Θ_0 or known in which case it equals $\hat{x}_0$ and $\Theta_0 = 0$.

To measure performance, we consider the following finite-horizon cost

$$\mathbb{E}[\sum_{k=0}^{N-1} (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k) + x_N^T Q_N x_N], \tag{6.2}$$

which is to be minimized, and where $Q, Q_N, R \succ 0$ are positive definite weighting matrices. By considering this performance index, the problem formulation differs form the one introduced in [63]. In fact, in [63] a discounted and an average cost were considered mainly for convenience, as for such costs we were able to provide a time-invariant policy for the triggering laws and control inputs. However, in practical applications with a certain objective to be met in a certain finite-time, finite-horizon costs may be more suitable, and hence, is considered here. As shown later in this section, (and to be expected) the scheduling and control input policy becomes in this case time-varying.

The network behavior can be modeled using the scheduling vector $\sigma_k = (\beta_k, \gamma_k) \in \{0,1\}^2, k \in \mathbb{S}$, in which $\beta_k = 1$ (or $\gamma_k = 1$) indicates the occurrence of a transmission through the network from sensor to controller (or controller to actuator) at time k and $\beta_k = 0$ (or $\gamma_k = 0$), otherwise. The vector u_k denotes the transmitted value of control actions at time $k \in \mathbb{S}$ and y_k represents the received value of sensor data at time $k \in \mathbb{S}$. When there is no transmission, we assume $u_k = \emptyset$ and $y_k = \emptyset$ indicating that no new information is received. At the actuator side, we consider a standard zero-order hold (ZOH) device as the control-input generator (CIG) to actuate the plant with the most recent received control action.

The ETC unit in Figure 6.1 controls the transmission decisions in both networks, i.e., network 1 between the sensors and the controller and network 2 between the controller and the actuators. Our solution will entail that for network 2, the controller needs only to send data when desired. However, for the network 1, the controller must first query the sensors and then receive measurement data. We assume that the delay introduced by this process is negligible and there are no packet drops in both networks.

Beside scheduling decisions, the ETC unit computes the control actions at transmission instants i.e., $\gamma_k = 1$ based on available information I_{k-1} at time k, which is defined as

$$I_k := (I_{k-1}, y_k, u_k, \sigma_k)$$

for $k \in \mathbb{S}$ and $I_{-1} := (\hat{x}_0,\ \Theta_0)$. A policy $\pi := (\mu_0, \mu_1, \ldots, \mu_{N-1})$ is defined as a sequence of functions $\mu_k := (\mu_k^u, \mu_k^\sigma)$ that map the available information vector I_{k-1} into control actions u_k and scheduling decisions σ_k in the sense that

$$(u_k, \sigma_k) = \mu_k(I_{k-1}). \tag{6.3}$$

for all $k \in \mathbb{S}$.

We denote by $J_\pi(I_{-1})$ the costs (6.2), when policy π is applied by the controller. Similar to Chapter 3, we are interested in a policy that reduces the number of transmissions compared to the all-time transmission policy, while keeping the performance within a desired bound of the performance of the all-time transmission policy. The all-time transmission policy is defined as $\sigma_k = (1,1)$ for every time step $k \in \mathbb{S}$, and an associated optimal policy for the control input.

We recall that such an optimal control input policy is given by

$$\mu^{u}_{all,k}(I_{k-1}) = L_k \hat{x}_{k|k-1}, \tag{6.4}$$

where $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ can be obtained by a time-varying Kalman filter, and

$$L_k = -(R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A,$$

where K_{k+1} denotes the solution of the discrete-time dynamic Ricatti equation (DDRE)

$$\begin{aligned} K_N &= Q_N \\ K_k &= Q + A^T K_{k+1} A - P_k \\ P_k &= A^T K_{k+1} B (R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A. \end{aligned} \tag{6.5}$$

The cost-to-go function of policy (6.4) (and $\sigma_k = (1,1),\ k \in \mathbb{S}$) denoted by π_{all} is

$$J_{\pi_{all}}(I_{k-1}) = \mathbb{E}\{x_k^T K_k x_k | I_{k-1}\} + \sum_{s=k+1}^{N} \mathrm{Tr}(K_s \Phi_v) + \sum_{s=k}^{N-1} \mathrm{Tr}(P_s \Sigma_s), \tag{6.6}$$

where $\Sigma_k = \mathrm{Cov}[x_k|I_{k-1}]$ denotes the conditional covariance matrix of the estimation error that can be expressed as

$$\Sigma_s = \mathrm{Ric}^{\,s}(\Theta_0), \quad s \in \mathbb{S}, \tag{6.7}$$

where

$$\mathrm{Ric}^{\,1}(\Sigma) = A\Sigma A^T + \Phi_v - A\Sigma C^T (C\Sigma C^T + \Phi_r)^{-1} C\Sigma A^T$$

and $\mathrm{Ric}^s = \mathrm{Ric}^{s-1} \circ \mathrm{Ric}^1$, $\mathrm{Ric}^0 = \mathrm{id}$.

Remark 6.1. By setting the penalty Q_N for the final state x_N equal to $\overline{K}$, where $\overline{K}$ is the steady state solution of DDRE (6.5), the DDRE becomes the discrete-time algebraic Ricatti equation (DARE) and the LQR-gain in (6.4) becomes time-invariant. Moreover, if the horizon is sufficiently large, the control policy for the first instances of time approaches this time-invariant policy corresponding to the infinite-horizon solution of the discrete-time algebraic Ricatti equation. Note that given (A, B) is controllable and Q and R are positive definite, the solution of the DDRE converges to that of the DARE as $N \to \infty$.

6.3 Control policy

The control policy validated in this chapter is parameterized by two non-negative scalars ζ, θ, and defined by

$$(u_k, \gamma_k) = \begin{cases} (L_k \hat{x}_{k|k-1}, 1), & \text{if } \eta_k^T \Gamma_k \eta_k > \lambda_k \\ (\emptyset, 0), & \text{otherwise} \end{cases} \tag{6.8}$$

and

$$\beta_k = \begin{cases} 1, & \text{if } h_k(\Sigma_k) > \zeta \\ 0, & \text{otherwise,} \end{cases} \tag{6.9}$$

where

$$\begin{aligned} \eta_k &= \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{u}_{k-1} \end{bmatrix} \\ \Gamma_k &= \begin{bmatrix} (1+\theta)P_k - \theta Q & (1+\theta)A^T K_k B \\ (1+\theta)B^T K_k A & R + (1+\theta)B^T K_k B \end{bmatrix} \\ \lambda_k &= \theta \operatorname{Tr}(Q\Sigma_k) \\ \hat{x}_{k|k-1} &= \mathbb{E}[x_k | I_{k-1}] \\ \Sigma_k &= \operatorname{Cov}[x_k | I_{k-1}] \end{aligned} \tag{6.10}$$

and

$$h_k(\Sigma_k) = \sum_{s=k+1}^{N} \operatorname{Tr}\Big(P_s\big(\operatorname{Ric}^{s}(A\Sigma_k A^T + \Phi_v) - \operatorname{Ric}^{s+1}(\Sigma_k)\big)\Big). \tag{6.11}$$

Theorem 6.1. Let $\theta \in \mathbb{R}_{\geq 0}$ and $\zeta \in \mathbb{R}_{\geq 0}$ be given. Consider system (6.1) with policy π (parameterized by θ and ζ) defined by (6.8)-(6.11). Then

$$J_\pi(I_{-1}) \leq (1+\theta)(J_{\pi_{all}}(I_{-1}) + N\zeta), \tag{6.12}$$

for every I_{-1}, where J_π refers to the cost (6.2) for the control policy π given by (6.8)-(6.11) and $J_{\pi_{all}}$ refer to the cost for the all-time transmission control policy given by (6.6).

The proof follows the same line of reasoning as the proof of Theorem 3.1 and is omitted for brevity.

Remark 6.2. Scalars θ and ζ can be used to balance the trade-off between guaranteed performance in terms of (6.2) and the number of transmissions. Clearly increasing ζ in (6.9) not only increases the guaranteed bound (6.12) but will also make the sensor-query triggering condition less stringent, which results in less transmissions from sensors to the controller. A similar reasoning can be applied to the parameter θ for the controller to actuator network. Note that since for $\Sigma \succeq 0$ and $h_k(\Sigma) \geq 0$, choosing $\zeta = 0$ results in a transmission in the sensor to controller network at (periodically) every time step $k \in \mathbb{S}$. Moreover, note that for $\theta = 0$, $\Gamma_k \succeq 0$, $k \in \mathbb{S}$ and $\lambda_k = 0$, which results in a transmission in the controller to actuator network at (periodically) every time step $k \in \mathbb{S}$. If $\zeta = 0$ and $\theta = 0$, we recover the all-time transmission control policy π_{all} and (6.12) holds with equality. Furthermore, the bound on the finite-horizon cost does not only depend on the parameters θ and ζ, but also on the length of the horizon N. Therefore, the horizon length can be seen as another tuning parameter to control the bound on the cost.

We show in the state estimate subsection of the current section that $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ and $\Sigma_k = \mathrm{Cov}[x_k|I_{k-1}]$ can be obtained by the controller by running the time-varying Kalman filter. As we shall see $\Sigma_k = \mathrm{Cov}[x_k|I_{k-1}]$ can be determined a priori, which entails that the scheduling sequence for sensor queries, triggered by condition (6.9), can be determined offline. In turn, the state estimate $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ depends on the noise realizations and, therefore, must be determined online. Consequently, the scheduling decisions, triggered by condition (6.8), must be determined by the controller online.

Remark 6.3. As mentioned in Remark 1, if we set penalty $Q_N = \overline{K}$ where $\overline{K}$ is the steady state solution of DDRE (6.5), the matrices Γ_k and scalars λ_k become time-invariant, which makes the proposed time-invariant similar to the policy in Chapter 3.

State estimate

The conditional distribution of x_k given I_{k-1} is Gaussian (the proof of this fact can be concluded from a similar proof in [84]) and therefore $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ and $\Sigma_k = \mathrm{Cov}[x_k|I_{k-1}]$ can be obtained by running the time-varying Kalman filter

$$\begin{aligned}\hat{x}_{k+1|k} &= A\hat{x}_{k|k-1} + B\hat{u}_k + \beta_k G_k\big(y_k - C\hat{x}_{k|k-1}\big)\\ G_k &= A\Sigma_k C^T(C\Sigma_k C^T + \Phi_r)^{-1}\\ \Sigma_{k+1} &= \overline{\mathrm{Ric}}(\Sigma_k, \beta_k),\end{aligned} \tag{6.13}$$

where

$$\overline{\mathrm{Ric}}(\Sigma, j) = A\Sigma A^T + \Phi_v - jA\Sigma C^T(C\Sigma C^T + \Phi_r)^{-1}C\Sigma A^T, \tag{6.14}$$

β_k is determined via (6.9) and $\hat{u}_k$ is the input to the plant, which is known to the controller

$$\hat{u}_k = \gamma_k L_k \hat{x}_{k|k-1} + (1-\gamma_k)\hat{u}_{k-1}. \tag{6.15}$$

6.4 Experimental set-up

The objective of the experiment is to control a robot in one direction (longitudinal). In particular, we consider a regulation problem by which the position of the robot is driven towards the zero position. The set-up for the experiments is depicted in Figure 6.2. The set-up consists of a robot, a wired camera, a control unit, which is the laptop in the figure, and a wireless network. The camera is used to obtain an image of the robot in its field of view. The camera takes an image at designated time instances, which are defined by the controller, and sends the images to the controller. The controller receives the images from the camera and processes the images to get the estimated position of the robot with respect to a certain reference frame placed in the environment. The camera is initially

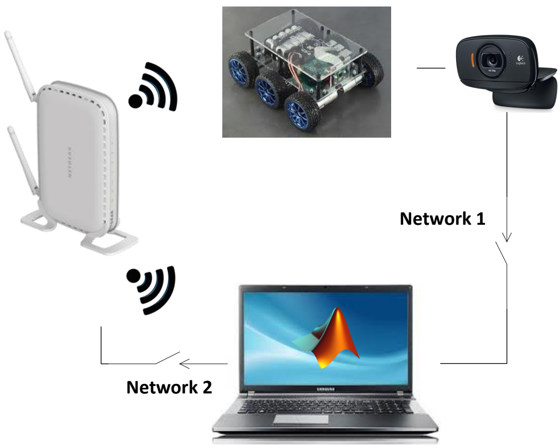


Figure 6.2. A simple block diagram of the experimental set-up.

calibrated based on given features of the environment with known positions with respect to this reference frame. Based on the estimated position of the robot, the controller is able to compute new control actions. These computed control actions are sent through a wireless network to the robot and are executed. In the sequel, the robot characteristics, the camera module, the controller and the communication network will be described.

Robot

The physical set-up we consider is a non-holonomic Diddyborg robot with six wheels as depicted in Figure 6.3. Each wheel of the robot is driven separately via a DC motor using a PWM-signal designed by the manufacturer. As the objective of the experiment is to control the robot in one dimension the same PWM-signal is used for all motors. Since all the motors are DC and the robot is only controlled in one direction the higher-order dynamics of the robot will be neglected. The dynamics of the robot can be modeled by the following continuous-time first-order linear system

$$\begin{aligned} \dot{x}(t) &= Bu(t) + v(t) \\ y(t) &= x(t) + r(t), \end{aligned} \tag{6.16}$$

where $x(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the position and the position measurement of the robot with respect to the given reference frame, and $u \in [-6V, 6V]$ denotes the voltage applied to the actuators at time $t \in \mathbb{R}_{\geq 0}$ denotes time. The state disturbance and measurement noise at time t are represented by $v(t)$ and $r(t)$, respectively.

Remark 6.4. The first-order system model has been identified using the Matlab

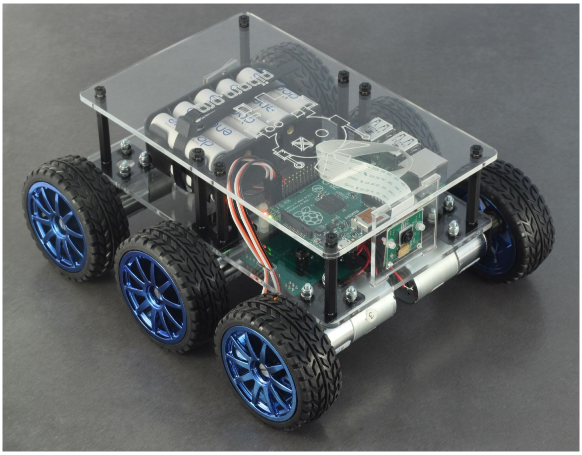


Figure 6.3. The Diddyborg robot used for the experiment.

identification toolbox, this lead to $B = 0.0023$. Given the sampling time T_s the discrete-time model of the system (6.16) can be described as

$$\begin{aligned} x_{k+1} &= x_k + T_s B u_k + v_k \\ y_k &= x_k + r_k, \end{aligned} \tag{6.17}$$

with $v_k \sim \mathcal{N}(0, \phi_v)$ and $r_k \sim \mathcal{N}(0, \phi_r)$ are assumed to be Gaussian zero-mean, $k \in \mathbb{S}$. The values of $\phi_v = 10^{-4}$ and $\phi_r = 10^{-6}$ are obtained through experiments.

Vision system and control unit

For the vision-based system the Logitech HD Webcam C525 is used, which is initially calibrated with the help of calibration cubes using the homogeneous transformation approach [100]. A pink marker is placed on the robot, in Figure 6.4 a HSV image of the robot is shown. On the control unit a Matlab program is running to estimate the position of the robot based on the image of the camera by the Perspective and Point approach (PnP) [100].

Remark 6.5. The detection algorithm requires computational power and is time-consuming. This justifies the use of the discrete-time model of the plant and imposes a lower bound on the sampling time, $T_s > 0.2$ seconds. Also note that this detection phase needs computational resources and, therefore, it is preferable to reduce the usage of the algorithm to compute the estimate of the state. This further emphasizes the need of a resource-allocation algorithm.

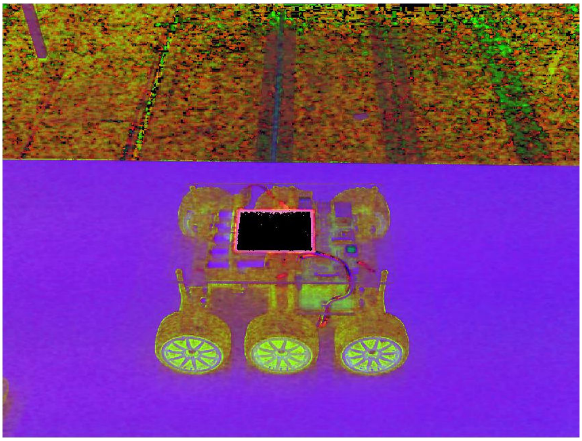


Figure 6.4. The pink marker detection in the HSV image.

Wireless communication

The communication between the robot and the controller is established through a wireless network provided by a hot-spot via a router. In order to facilitate the transmission of the control input, we use unified datagram protocol (UDP).

Remark 6.6. UDP is a lightweight simple protocol. We choose this protocol because of its simplicity and its fast execution time. During the experiments we did not encounter any problems regarding packet drops or transmission delays. In fact, the delays are negligible w.r.t. our sampling time. So, for our validation the UDP protocol is adequate.

Experiment

In the experiment, the motion system is controlled in one direction (longitudinal). The robot is regulated to the zero position from an approximate initial position of 0.7 m. A horizon of $N = 60$ time steps is considered, since for the control policies this horizon is sufficiently large to regulate the robot to the zero position. The horizon of $N = 60$ time steps corresponds to 18 seconds since the sampling time T_s is set to 0.3 seconds. The values of the weighting matrices Q, Q_N and R are tuned to obtain a smooth performance in the robot movement an selected as

$$Q = 10 \quad Q_N = 200, \quad R = 0.003.$$

We compare two cases, one with $\theta = \zeta = 0$, which corresponds to an all-time transmission policy π_{all}, and the other one with $\theta = 0.5$ and $\zeta = 0.004$ implementing the policy π as in (6.8)-(6.9). The state estimation and related covariances are computed using a time-varying Kalman filter, see (6.13).

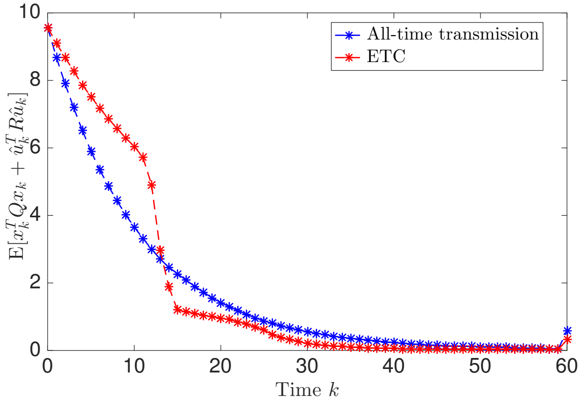


Figure 6.5. $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ estimated via 100 Monte Carlo simulations in two cases $\theta = \zeta = 0$ and $\theta = 0.5$, $\zeta = 0.004$.

6.5 Numerical results

In this section, we present the results of numerical simulations based on the obtained model (6.17). By applying the ETC scheme, the sensor to controller and the controller to actuator networks usage are reduced by 80% and 90% respectively. Figure 6.5 shows the estimated running cost $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ based on Monte-Carlo simulations for both cases. The cost of the ETC policy obtained via Monte-Carlo simulations satisfies the performance bound (6.12), which for this case becomes

$$113.15 = J_{\pi}^{sim} \leq 1.5(J_{\pi_{all}}^{sim} + 60 \star 0.004) = 158.85.$$

where $J_{\pi_{all}}^{sim} = 103.66$. Notice that the cost of the ETC method is significantly less than the theoretical performance bound and close to the cost of the all-time transmission policy $J_{\pi_{all}}^{sim}$. This indicates that the bound is conservative and performance can be much better than the actual guaranteed upper bound. To see the reduction in communications, the actuator signals of one realization of both cases with the same noise are shown in Figure 6.6, the values of γ_k for the ETC policy are also shown. As can be seen, the actuator holds its value more often (i.e., less transmissions occur) when applying the ETC method. The evolution of the trajectory $h_k(\Sigma_k)$ is shown in Figure 6.7. At the time instances where $h_k(\Sigma_k)$ exceeds the threshold ζ, the control unit acquires a new image from the camera through the sensor network. The evolution of the trajectory of $h_k(\Sigma_k)$ exhibits, once every five time steps $h_k(\Sigma_k)$ exceeds ζ and, thus, β_k becomes 1.

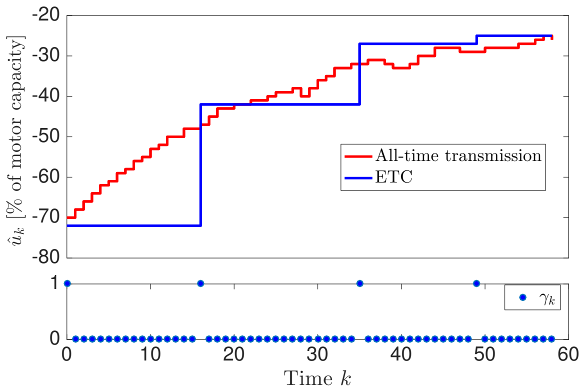


Figure 6.6. Trajectory of actuation signal, $\hat{u}_k$, for two cases with $\theta = \zeta = 0$ and $\theta = 0.5, \zeta = 0.004$ obtained by simulation. For $\gamma_k = 1$ a new control input $\hat{u}_k$ for the ETC policy is computed.

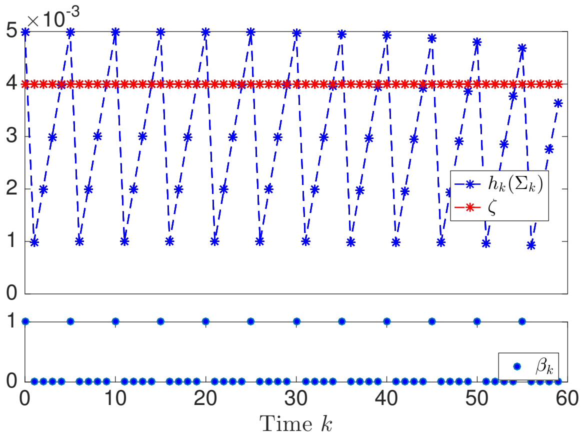


Figure 6.7. The offline computed evaluation of $h_k(\Sigma_k)$ for the case $\theta = 0.5$ and $\zeta = 0.004$. For $\beta_k = 1$ a new image is sent from the camera over the network to the control unit.

6.6 Experimental results

In this section, we present the experimental results. The settings for the experiments are already discussed in Section 6.4. For both control policies the experiment is carried out 10 times with similar initial conditions. Since the settings for the experiment are the same as for the numerical simulations, the behavior of the sensor to controller network is the same for both the simulations and the experiment. The a priori calculated evaluation of $h_k(\Sigma_k)$ for the ETC policy is shown in Figure 6.7. The communication in the sensor to controller

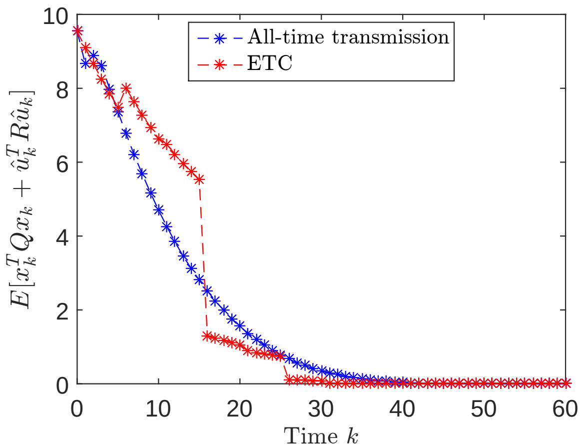


Figure 6.8. The experimental running cost $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ for two cases with $\theta = \zeta = 0$ (all-time transmission) and $\theta = 0.5, \zeta = 0.004$.

network is reduced by 80% compared to the all-time transmission control policy. For the experiments carried out the communication in the actuator network is reduced by 90%. In Figure 6.8, the running cost $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$, $k \in \mathbb{S}$ for both the all-time transmission and the ETC policy are shown (the average is taken over all 10 experiments). The total expected cost for the ETC policy satisfies the performance bound on the cost (6.12), which for the experimental set-up yields

$$127.85 = J_{\pi}^{exp} \leq 1.5(J_{\pi_{all}}^{exp} + 60 \cdot 0.004) = 175.06.$$

where $J_{\pi_{all}}^{exp} = 116.47$ is the expected cost for running the all-time transmission policy. Notice that the cost of the ETC method is much less than the performance bound and close to the cost of the all-time transmission policy $J_{\pi_{all}}^{exp}$. Again we see that the bound is conservative. In Figure 6.9, the control inputs $\hat{u}$ for both the periodic control and the ETC policy are shown for a single experiment, also the value of γ_k for the ETC policy is shown. For the ETC policy, a control input is only sent five times. This is clearly visible in Figure 6.9. This result shows the advantage of the applied ETC policy. Only if necessary, the actuator network is used and otherwise the resources are saved. In Figure 6.10 the estimated position of the robot, $\hat{x}_k$, $k \in \mathbb{S}$ is shown for both the periodic control policy and the ETC policy. In the considered horizon of $N = 60$ time steps the robot is for both policies regulated from its initial position to the zero position. The estimated state for the ETC policy, in Figure 6.10, shows a jump every time $\beta_k = 1$ (as long as the state is not yet regulated to zero). This jump is due to the fact that the estimator does not receive any measurement data for 5 time steps and therefore the estimation is purely model-based. Consequently,

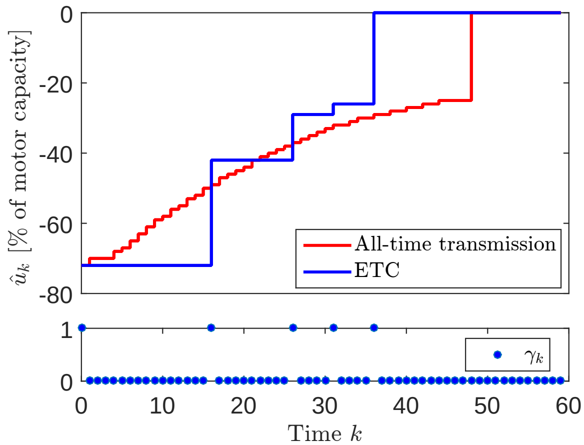


Figure 6.9. Trajectory of the actuation signal in the experimental set-up, $\hat{u}_k$, for a single experiment for two cases with $\theta = \zeta = 0$ and $\theta = 0.5, \zeta = 0.004$. For $\gamma_k = 1$ a new control input $\hat{u}_k$ is computed.

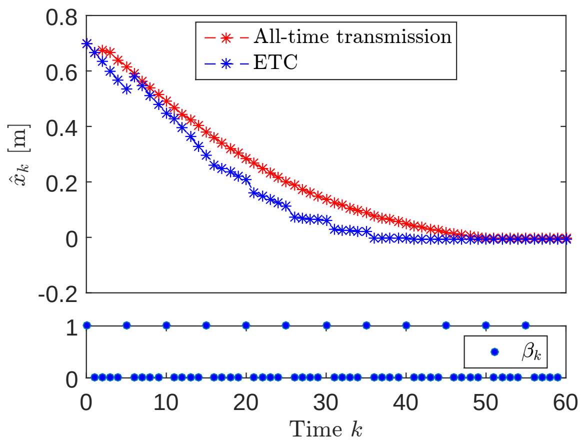


Figure 6.10. The estimated position $\hat{x}_k$ of the real motion system for two cases with $\theta = \zeta = 0$ (all-time transmission) and $\theta = 0.5, \zeta = 0.004$. For $\beta_k = 1$ a new image is sent from the camera over the network to the control unit.

receiving a new image correct the estimation based on measurement data and, therefore, leads to a visible jump in the state estimate. The estimated state for the all-time transmission policy does not have such large jumps since the estimated state is updated each time step with a new image from the camera.

6.7 Numerical versus experimental results

In this section, the numerical results will be compared with the experimental results. Some interesting observations can be pointed out. The expected cost obtained by Monte-Carlo simulations is relatively close to the expected cost obtained in the experiments for both control policies. The costs in the simulations are slightly lower. In the control policy the disturbances are assumed to be Gaussian zero-mean, this does not perfectly match the reality. This can cause a slightly higher cost in the experiments compared to the cost in the simulations. The behavior of the expected running cost is comparable for the simulations and the experiments, see Figure 6.5 and Figure 6.8. In the expected running cost for the ETC policy for the experiments in Figure 6.8, there is a jump visible at $k = 5$. This jump is due to a mismatch between the model of the robot and the actual dynamics of the robot, as already explained at end of Section 6.6.

The network utilization shows similar results for the simulations and the experiments. The behavior of the sensor network is computed offline and, therefore, the same for both cases. However, in the controller to actuator network the scheduling mechanism is online and interestingly we see similar results. In the simulations, the communication is reduced by 90% reduction and in the experiments the communication is reduced by 90%. This further validates the effectiveness of the proposed control policy.

6.8 Conclusions

In this chapter, we validated (a finite-horizon version of) the ETC strategy for linear discrete-time systems derived in Chapter 3. The experiment validated that the proposed ETC policy significantly reduces the communication in the sensor to controller and the controller to actuator networks, while guaranteeing performance bounds on the cost. From the validation we have learned that for one-dimensional movement, communication can be reduced by 80% and 90% for the sensor to controller and the controller to actuator networks, respectively, compared with the all-time transmission control policy (while only sacrificing a bit of performance). Small differences between the simulations and the experiments can be explained by the fact that in the control policy the disturbances are assumed to be Gaussian zero-mean, which does not perfectly match the reality. An interesting observation of the policy is that the guarantees concerning the performance bound are conservative. Hence, the ETC might performs much better than the guarantees provided. This is also the case as the cost for the ETC policy is relatively close to the cost for the all-time transmission policy (despite the large gap on the guaranteed performance).

The validated policy in this chapter can not deal with packet drops and transmission delays. However, in Chapter 7, we propose a suboptimal event-triggered policy for unreliable networks links including performance guarantees.

Moreover, we implement this new ETC policy on a more challenging omni-directional robot regulating the robot towards a predefined trajectory in a 2D space.

Chapter 7

Suboptimal Event-Triggered Control over Unreliable Communication Links with Experimental Validation

In this chapter[1], we propose an ETC policy for a discrete-time linear system with unreliable actuators' and sensors' links, captured by Bernoulli packet dropout models. The proposed policy is an absolute-threshold policy by which transmissions occur if a weighted norm of an error state vector exceeds a threshold. The threshold and the weights of the norm depend on the underlying characteristics of the packet droupout model. Such a policy is shown to guarantee a given performance, defined in terms of a quadratic cost, while reducing transmissions. The proposed ETC policy is experimentally validated in the context of remotely steering an omni-directional ground robot along a predefined trajectory over an unreliable wireless network, while keeping transmissions to a minimum.

7.1 Introduction

In the design of ETC, it is important to incorporate network artifacts such as packet dropouts, packet delays, and quantization errors, as these are present in almost all practical applications. This asks for the adaptation of the body of work in ETC to take into account these network induced artifacts. The work [101] considers the optimal control policy design for scalar systems with limited control actions under unreliable actuator links. It is shown that the optimal policy

[1]This chapter is based on [69].

is an ETC policy in which the new control input is transmitted to the actuator if the norm of the state estimate is greater than a threshold. This threshold is obtained through a look-up table that depends on the knowledge of whether or not a dropout has happened and is determined by enumerating all possible scenarios. In [102], the triggering policy consists of a deterministic threshold-based policy and a probabilistic network access protocol determining which agent gets access to the shared network. There it was shown that if the triggering policy is aware of whether or not a packet has been dropped then the proposed policy is robust in the sense that the quadratic measure of the aggregate error remains bounded in expectation. In [103], the control of scalar Brownian motion under delta sampling, a special class of level-triggered sampling, is investigated. Furthermore, it is shown that the distortion of the deterministic sampler is always higher than the distortion due to the delta sampling. In [104], a combination of an event-based predictive control and a network compensator is shown to provide closed-loop stability of nonlinear continuous-time systems in the presence of packet dropouts. In [105], by introducing more realistic models for the communication channels with packet dropouts and time-delay, it has been shown that certainty equivalence is still optimal if an instantaneous error-free acknowledgment channel exists. In [58], utilizing a performance index in terms of the second moment of a scalar stochastic linear system, an ETC mechanism is designed such that, in the presence of packet dropouts, the second moment of the state converges exponentially to a desired set in finite time. It is further shown that the proposed policy in [58] under mild conditions provides guaranteed bounds over the transmission rate. In [85] the design of an output-based dynamic event-triggered mechanism for nonlinear systems subject to packet losses both with and without acknowledgment is studied. It was shown that the proposed mechanism can admit a maximum allowable number of successive packet dropouts, while still maintaining the desired stability and performance properties.

In this chapter, we focus on the development of ETC policies that provide guaranteed closed-loop performance in the presence of packet losses. To this effect, we assume that the network links connecting the sensors to the controller and the controller to actuators are subject to failures (see Figure 7.1). In this context, we provide policies that take into account two different information structures at the controller side: one without any receipt acknowledgment of actuation signal packets to the controller referred to as UDP-like protocols, and the other with receipt acknowledgment of actuation signal packets referred to as TCP-like protocol [106]. We show that, taking into account a quadratic performance index, we can design a threshold event-triggered mechanism based on a weighted norm of the state estimate such that its performance is within a predefined neighborhood of the all-time transmission policy proposed in [106]. Furthermore, the applicability of one of the proposed policies is validated through experimental results. The experimental set-up consists of an omni-directional robot a top camera above the (soccer) field, where the robot is moving, and a

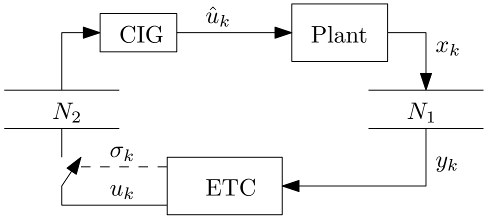


Figure 7.1. Considered control structure. The sensor network, N_1, and the actuator network N_2 are subject to Bernoulli dropout failures. CIG stands for control-input generator.

controller on a host PC. Images of the robot on the soccer field are sent from the top camera to the controller via a LAN network. These images are processed to obtain the position and orientation of the robot with respect to the world frame. Then, the remote controller steers the robot along a predefined trajectory by transmitting the computed control actions to the robot via a wireless network with a UDP-like protocol.

The structure of the remainder of the chapter is as follows. In Section 7.2 we formulate the problem, in Section 7.3 we explain the proposed policies. Section 7.4 provides simulation results and in Section 7.5 the experimental setup is introduced and the results of the experiments are provided. Section 7.6 concludes the chapter.

7.2 Control Structure

A schematic of the control setup is depicted in Figure 7.1. The plant is a linear discrete-time dynamical system

$$x_{k+1} = Ax_k + B\hat{u}_k + v_k, \quad k \in \mathbb{S}, \tag{7.1}$$

where $\mathbb{S} = \{0, 1, \dots, N-1\}$, $N \in \mathbb{N}$ and $x_k \in \mathbb{R}^{n_x}$, $\hat{u}_k \in \mathbb{R}^{n_u}$ are the state and the actuation signal, respectively. Moreover, $(v_k)_{k=0}^{N-1}$ is a sequence of Gaussian i.i.d. random vectors with zero mean and covariance Φ_v. We consider a network between the sensors and the controller N_1, and a network between the controller and the actuators N_2. In a similar setting as in [106, 107], we assume that the networks are lossy with dropout probabilities α and β, respectively. The control unit not only computes the control action u_k but also decides whether or not to transmit the computed control action based on the available information. This scheduling is modeled via a decision variable $\sigma_k \in \{0, 1\}$, $k \in \mathbb{S}$, where $\sigma_k = 1$ denotes the occurrence of a transmission and $\sigma_k = 0$ a non-transmission event. At the actuator side, we consider a control-input-generator (CIG) that sets the actuation signal to zero when no control input is received from the controller. Let $(\alpha_k)_{k\in\mathbb{S}}$ and $(\beta_k)_{k\in\mathbb{S}}$ be i.i.d. Bernoulli processes which model the unreliable nature of the link from the controller to the actuators and sensors to controller, respectively. Then,

$$\hat{u}_k = \alpha_k \sigma_k u_k, \quad k \in \mathbb{S}, \tag{7.2}$$

and the system (7.1) can be modeled as

$$\begin{aligned} x_{k+1} &= Ax_k + \alpha_k \sigma_k B u_k + v_k \\ y_k &= \begin{cases} x_k, & \beta_k = 1, \\ \emptyset, & \text{otherwise.} \end{cases} \end{aligned} \tag{7.3}$$

The packet dropouts have the following probability distribution

$$\begin{aligned} P[\alpha_k = 1] &= 1 - \alpha = \bar{\alpha} \\ P[\alpha_k = 0] &= \alpha \end{aligned} \tag{7.4}$$

and

$$\begin{aligned} P[\beta_k = 1] &= 1 - \beta = \bar{\beta} \\ P[\beta_k = 0] &= \beta. \end{aligned} \tag{7.5}$$

We assume that $(\alpha_k)_{k \in \mathbb{S}}$ and $(\beta_k)_{k \in \mathbb{S}}$ are independent processes, also independent of the disturbance process $(v_k)_{k \in \mathbb{S}}$, and the initial state x_0. Moreover, note that $y_k \in \mathbb{R}^{n_x}$ denotes the accessible state of the plant to the controller and the symbol $y_k = \emptyset$ is used to denote the absence of information when a packet drop occurs.

To design and analyze the controller, we consider the following finite-horizon quadratic performance criterion

$$J = \mathbb{E}[x_N^T Q_N x_N + \sum_{k=0}^{N-1} x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k], \tag{7.6}$$

where Q and R are positive definite weighting matrices of proper dimensions. To penalize transmissions, we follow a similar approach as in [49] and introduce an additive penalty term to the performance index (7.6). Thus we obtain

$$V = \mathbb{E}[x_N^T Q_N x_N + \sum_{k=0}^{N-1} x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k + \lambda \sigma_k], \tag{7.7}$$

where $\lambda > 0$ characterizes the penalty on transmissions.

We define a control and scheduling policy π as a set of functions

$$\pi = \{(\mu_0^\sigma(I_0), \mu_0^u(I_0)), \ldots, (\mu_{N-1}^\sigma(I_{N-1}), \mu_{N-1}^u(I_{N-1}))\}. \tag{7.8}$$

In particular, the available information vector I_k (to be specified) at time $k \in \mathbb{S}$, is mapped to the scheduling variable and control action (σ_k, u_k) as

$$(\sigma_k, u_k) = \mu_k(I_k) = (\mu_k^\sigma(I_0), \mu_k^u(I_k)). \tag{7.9}$$

We consider three different scenarios for the information vector:

1. **Ideal networks:** in this scenario both networks are ideal with no packet dropouts and therefore the information set can be written as

$$
\begin{aligned}
&I_k^I = (I_{k-1}^I, x_k, u_{k-1}, \sigma_{k-1}), \quad k \in \mathbb{S}\backslash\{0\} \\
&I_0^I = x_0.
\end{aligned} \tag{7.10}
$$

2. **TCP-like networks:** in this scenario, we assume that both networks are unreliable and they use TCP-like protocols for communication. The main feature of TCP-like communication protocols is that a binary acknowledgment is sent over the reverse link to the controller, whenever a packet has been received successfully by the CIG. It is assumed that the reverse link is error-free which result in the information set

$$
\begin{aligned}
&I_k^{TCP} = (I_{k-1}^{TCP}, y_k, \beta_k, u_{k-1}, \sigma_{k-1}, \alpha_{k-1}), k \in \mathbb{S}\backslash\{0\} \\
&I_0^{TCP} = (y_0, \beta_0).
\end{aligned} \tag{7.11}
$$

3. **UDP-like networks:** in this scenario, both networks are unreliable and a UDP-like protocol in which there is no acknowledgment signal is applied. The information set can be represented as

$$
\begin{aligned}
&I_k^{UDP} = (I_{k-1}^{UDP}, y_k, \beta_k, u_{k-1}, \sigma_{k-1}), \quad k \in \mathbb{S}\backslash\{0\} \\
&I_0^{UDP} = (y_0, \beta_0).
\end{aligned} \tag{7.12}
$$

We denote the average triggering rate as

$$
R_t = \frac{1}{N}\,\mathbb{E}[\sum_{k=0}^{N-1} \sigma_k]. \tag{7.13}
$$

The optimal control and scheduling policy design focuses on finding the policy π_{opt} to minimize (7.7). This problem is a mixed-integer programing and therefore obtaining an optimal solution is computationally not tractable. Therefore, in this chapter, we focus on finding a *good* policy π that is simple to implement and achieves a performance in terms of (7.6), which is within a guaranteed analytical bound. We propose three control policies corresponding to the three aforementioned information structures. The proposed policies are built upon the roll-out algorithm (see, e.g., [71]) in the context of approximate dynamic programming considering a base triggering policy of all-time transmission $\sigma_k = 1,\ k \in \mathbb{S}$, and optimal control policy. The provided guaranteed analytical bounds are obtained using the performance of this base policy.

7.3 ETC for unreliable communication links

ETC for UDP-like communication

We assume that both actuator and sensor network links are prone to failure, i.e., $\beta > 0,\ \alpha > 0$, and there is no acknowledgment of successful transmission in the actuator network. Then the available information at the controller is characterized by (7.12). In this setting, if transmissions are triggered at all time instants i.e. $\sigma_k = 1,\ k \in \mathbb{S}$, then the problem reduces to the optimal control design investigated in [106]:

$$\begin{aligned}
u_k &= L_k \hat{x}_k \\
L_k &= -(R + B^T(K_{k+1} + \alpha\beta P_{k+1})B)^{-1} B^T K_{k+1} A \\
P_k &= \bar{\alpha} A^T K_{k+1} B (R + B^T(K_{k+1} + \alpha\beta P_{k+1})B)^{-1} \\
&\quad \times B^T K_{k+1} A + \beta A^T P_{k+1} A \\
K_k &= A^T K_{k+1} A - P_k + \beta A^T P_{k+1} A + Q,
\end{aligned} \tag{7.14}$$

where $K_N = Q_N,\ P_N = 0$ and $e_k := x_k - \hat{x}_k$ with $\hat{x}_k$ the state estimate whose dynamics are governed by the optimal estimator (see, [106, eq. 17]),

$$\hat{x}_k = \begin{cases} A\hat{x}_{k-1} + \bar{\alpha} B u_{k-1} & \text{if } \beta_k = 0 \\ x_k & \text{if } \beta_k = 1. \end{cases} \tag{7.15}$$

Lemma 7.1. The control policy (7.14), (7.15) for all-time transmission with a UDP-like network protocol results in the overall performance of

$$\begin{aligned}
V_0^{\pi_{all}}(I_0^{UDP}) =& \mathbb{E}[x_0^T K_0 x_0 | I_0^{UDP}] + \mathbb{E}[e_0^T P_0 e_0 | I_0^{UDP}] \\
&+ \sum_{i=0}^{N-1} \mathbb{E}[v_i^T (K_{i+1} + \beta P_{i+1}) v_i] + N\lambda \\
=& J_0^{\pi_{all}}(I_0^{UDP}) + N\lambda,
\end{aligned} \tag{7.16}$$

where

$$J_0^{\pi_{all}}(I_0^{UDP}) = \mathbb{E}[x_0^T K_0 x_0 | I_0^{UDP}] + \mathbb{E}[e_0^T P_0 e_0 | I_0^{UDP}] + \sum_{i=0}^{N-1} \mathbb{E}[v_i^T (K_{i+1} + \beta P_{i+1}) v_i]. \tag{7.17}$$

Proof. We use a dynamic programming formulation for calculation the cost-to-go of the policy (7.14). By definition, the cost-to-go of the last stage is $V_N(I_N^{UDP}) = \mathbb{E}[x_N^T K_N x_N]$. Therefore, applying the policy $u_{N-1} = L_{N-1}\hat{x}_{N-1}$ we obtain the cost-to-go at stage $N-1$ as

$$V_{N-1}(I_{N-1}^{UDP}) = \mathbb{E}[x_{N-1}^T K_{N-1} x_{N-1} | I_{N-1}^{UDP}] + \mathbb{E}[e_{N-1}^T P_{N-1} e_{N-1} | I_{N-1}^{UDP}] \\ + \mathbb{E}[v_{N-1}^T K_N v_{N-1}] + \lambda. \quad (7.18)$$

For the next stage first note that based on (7.15), the estimation error has the dynamics

$$e_k = \begin{cases} Ae_{k-1} + (\alpha_k - \bar{\alpha})Bu_{k-1} + v_{k-1}, & \text{if } \beta_k = 0 \\ 0, & \text{if } \beta_k = 1. \end{cases} \quad (7.19)$$

Therefore, we obtain

$$\mathbb{E}[e_{N-1}^T P_{N-1} e_{N-1} | I_{N-2}^{UDP}] = \beta\,\mathbb{E}[e_{N-2}^T A^T P_{N-1} A e_{N-2} | I_{N-2}^{UDP}] \\ + \bar{\alpha}\alpha\beta\,\mathbb{E}[u_{N-2}^T B^T P_{N-1} B u_{N-2}] + \beta\,\mathbb{E}[v_{N-2}^T P_{N-1} v_{N-2}], \quad (7.20)$$

where we used

$$\mathbb{E}[(\alpha_k - \bar{\alpha})^2] = \bar{\alpha}(1-\bar{\alpha})^2 + (1-\bar{\alpha})\bar{\alpha}^2 = \bar{\alpha}(1-\bar{\alpha}) = \bar{\alpha}\alpha.$$

If we apply the control input $u_{N-2} = L_{N-2}\hat{x}_{N-2}$, then the dynamic programming formulation results in

$$V_{N-2}(I_{N-2}^{UDP}) = \mathbb{E}[x_{N-2}^T K_{N-2} x_{N-2} | I_{N-2}^{UDP}] + \mathbb{E}[e_{N-2}^T P_{N-2} e_{N-2} | I_{N-2}^{UDP}] \\ + \mathbb{E}[v_{N-2}^T (K_{N-1} + \beta P_{N-1}) v_{N-2}] + \mathbb{E}[v_{N-1}^T K_N v_{N-1}] + 2\lambda. \quad (7.21)$$

Proceeding similarly at stages $N-3,\ N-4,\ \ldots,\ 0$, we obtain the overall cost as in (7.16). □

In the following theorem, the proposed ETC policy under the information structure (7.12) is introduced. The goal of the ETC design is to find a policy that provides a guaranteed performance under unreliable communication links, while reducing the number of transmissions in the actuator network.

Theorem 7.1. Consider the system (7.3) with control and scheduling policy π defined as

$$(u_k, \sigma_k) = \begin{cases} (L_k \hat{x}_k, 1), & \text{if } \hat{x}_k^T \Gamma_k \hat{x}_k > \lambda \\ (\emptyset, 0), & \text{otherwise} \end{cases} \quad (7.22)$$

$$\Gamma_k = \bar{\alpha} A^T K_{k+1} B (R + B^T (K_{k+1} + \alpha\beta P_{k+1}) B)^{-1} \\ \times B^T K_{k+1} A,$$

where

$$\hat{x}_k = \begin{cases} A\hat{x}_{k-1} + \bar{\alpha} B \sigma_{k-1} u_{k-1} & \text{if } \beta_k = 0 \\ x_k & \text{if } \beta_k = 1. \end{cases} \quad (7.23)$$

Then the cost of the policy π, denoted by $J_0^{\pi}(I_0^{UDP})$, satisfies

$$J_0^{\pi}(I_0^{UDP}) \leq J_0^{\pi_{all}}(I_0^{UDP}) + N\lambda. \tag{7.24}$$

Proof. We apply the concept of roll-out strategies in the context of policy improvement algorithms [71] with a base policy of all-time transmission. If we apply the triggering (7.22) at time step $k \in \mathbb{N}_0$ assuming that a base policy of all-time transmission is applied afterwards, then we obtain the following cost

$$W_k(I_k^{UDP}) = \mathbb{E}[x_k^T Q x_k + \sigma_k \alpha_k u_k^T R u_k + \sigma_k \lambda + V_{k+1}^{\pi_{all}}(I_{k+1}^{UDP}) | I_k^{UDP}], \tag{7.25}$$

where

$$\begin{aligned} V_k^{\pi_{all}}(I_k^{UDP}) = \mathbb{E}[x_k^T K_k x_k | I_k^{UDP}] + \mathbb{E}[e_k^T P_k e_k | I_k^{UDP}] \\ + \sum_{i=k}^{N-1} \mathbb{E}[v_i^T (K_{i+1} + \beta P_{i+1}) v_i] + (N-k)\lambda. \end{aligned} \tag{7.26}$$

If $\lambda < \hat{x}_k^T \Gamma_k \hat{x}_k$, then $\sigma_k = 1$ and applying the control $L_k \hat{x}_k$ results in

$$W_k(I_k^{UDP}) = V_k^{\pi_{all}}(I_k^{UDP}).$$

On the other hand, if $\lambda \geq \hat{x}_k^T \Gamma_k \hat{x}_k$, then $\sigma_k = 0$, which leads to $x_{k+1} = A x_k + v_k$ and hence,

$$\begin{aligned} W_k(I_k^{UDP}) &= \mathbb{E}[x_k^T Q x_k + V_{k+1}^{\pi_{all}}(I_{k+1}^{UDP}) | I_k^{UDP}] \\ &= \mathbb{E}[x_k^T (Q + A^T K_{k+1} A) x_k | I_k^{UDP}] + \beta\, \mathbb{E}[e_k^T A^T P_{k+1} A e_k | I_k^{UDP}] \\ &\quad + \mathbb{E}[v_k^T (K_{k+1} + \beta P_{k+1}) v_k] + (N-k-1)\lambda \\ &= \mathbb{E}[x_k^T K_k x_k | I_k^{UDP}] + \mathbb{E}[x_k^T \Gamma_k x_k | I_k^{UDP}] + \mathbb{E}[e_k^T P_k e_k | I_k^{UDP}] \\ &\quad - \mathbb{E}[e_k^T \Gamma_k e_k | I_k^{UDP}] + \mathbb{E}[v_k^T (K_{k+1} + \beta P_{k+1}) v_k] + (N-k-1)\lambda \\ &= V_k^{\pi_{all}}(I_k^{UDP}) - \lambda + \hat{x}_k^T \Gamma_k \hat{x}_k \\ &\leq V_k^{\pi_{all}}(I_k^{UDP}), \end{aligned}$$

where we used

$$\mathbb{E}[x_k^T \Gamma_k x_k | I_k^{UDP}] - \mathbb{E}[e_k^T \Gamma_k e_k | I_k^{UDP}] = \hat{x}_k^T \Gamma_k \hat{x}_k,$$

and $\lambda \geq \hat{x}_k^T \Gamma_k \hat{x}_k$. Therefore, by applying policy (7.22) we obtain

$$W_k(I_k^{UDP}) \leq V_k^{\pi_{all}}(I_k^{UDP}). \tag{7.27}$$

Consequently, by using induction arguments as in [71, p. 338] we get

$$V_0^{\pi}(I_0^{UDP}) \leq V_0^{\pi_{all}}(I_0^{UDP}).$$

Moreover, based on the definition of the performance indexes we have $J_0(I_0) \leq V_0(I_0)$ for any policy, and therefore, using (7.16) we obtain

$$J_0^{\pi}(I_0^{UDP}) \leq J_0^{\pi_{all}}(I_0^{UDP}) + N\lambda.$$

□

ETC for TCP-like communication

A variant of the considered control problem is when a binary acknowledgment is sent over the reverse link to the controller, whenever a packet has been received successfully by the CIG. This will change the available information to the controller to TCP-like information structure (7.11). However, a similar procedure for designing of the ETC policy can be followed.

First we introduce the optimal all-time (i.e. with $\sigma_k = 1$, $k \in \mathbb{S}$) control policy, which minimizes (7.7) under information structure (7.11) (see, [106])

$$\begin{aligned} u_k &= L_k \hat{x}_k \\ L_k &= -(R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A \\ K_k &= -\bar{\alpha} A^T K_{k+1} B (R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A \\ &\quad + Q + A^T K_{k+1} A, \end{aligned} \tag{7.28}$$

with $K_N = Q_N$ and the optimal estimator is governed by

$$\hat{x}_k = \begin{cases} A\hat{x}_{k-1} + \alpha_{k-1} B u_{k-1}, & \text{if } \beta_k = 0 \\ x_k, & \text{if } \beta_k = 1. \end{cases} \tag{7.29}$$

Lemma 7.2. The control policy (7.28), (7.29) for all-time transmission with a TCP-like network protocol results in the overall performance of

$$\begin{aligned} V_0^{\pi_{all}}(I_0^{TCP}) = &\mathbb{E}[x_0^T K_0 x_0 | I_0^{TCP}] + \mathbb{E}[e_0^T P_0 e_0 | I_0^{TCP}] \\ &+ \sum_{i=0}^{N-1} \mathbb{E}[v_i^T (K_{i+1} + \beta P_{i+1}) v_i] + N\lambda, \end{aligned} \tag{7.30}$$

where

$$\begin{aligned} P_k &= \bar{\alpha} A^T K_{k+1} B (R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A \\ &\quad + \beta A^T P_{k+1} A, \end{aligned} \tag{7.31}$$

with $P_N = 0$.

Proof. We use a dynamic programming formulation for calculation the cost-to-go of the policy (7.28). By definition, the cost-to-go of the last stage is $V_N(I_N^{TCP}) = \mathbb{E}[x_N^T K_N x_N]$. Therefore, applying the policy $u_{N-1} = L_{N-1}\hat{x}_{N-1}$ we obtain the cost-to-go at stage $N-1$ as

$$\begin{aligned} V_{N-1}(I_{N-1}^{TCP}) = &\mathbb{E}[x_{N-1}^T K_{N-1} x_{N-1} | I_{N-1}^{TCP}] \\ &+ \mathbb{E}[e_{N-1}^T P_{N-1} e_{N-1} | I_{N-1}^{TCP}] \\ &+ \mathbb{E}[v_{N-1}^T K_N v_{N-1}] + \lambda. \end{aligned} \tag{7.32}$$

For the next stage, if we apply the control input $u_{N-2} = L_{N-2}\hat{x}_{N-2}$, the dynamic programming formulation results in

$$\begin{aligned} V_{N-2}(I_{N-2}^{TCP}) = \mathbb{E}[x_{N-2}^T K_{N-2} x_{N-2} | I_{N-2}^{TCP}] \\ + \mathbb{E}[e_{N-2}^T G_{N-2} e_{N-2} | I_{N-2}^{TCP}] \\ + \mathbb{E}[v_{N-2}^T K_{N-1} v_{N-2}] \\ + \mathbb{E}[v_{N-1}^T K_N v_{N-1}] \\ + \mathbb{E}[e_{N-1}^T P_{N-1} e_{N-1} | I_{N-2}^{TCP}] + 2\lambda, \end{aligned} \tag{7.33}$$

where

$$G_k = \bar{\alpha} A^T K_{k+1} B (R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A. \tag{7.34}$$

Now we focus on the last term in the right-hand side of equation (7.33). Note that based on (7.29), the estimation error has the dynamics

$$e_k = \begin{cases} A e_{k-1} + v_{k-1}, & \text{if } \beta_k = 0 \\ 0, & \text{if } \beta_k = 1. \end{cases} \tag{7.35}$$

This results in

$$\begin{aligned} \mathbb{E}[e_{N-1}^T P_{N-1} e_{N-1} | I_{N-2}^{TCP}] = \\ \beta\, \mathbb{E}[e_{N-2}^T A^T P_{N-1} A e_{N-2} | I_{N-2}^{TCP}] \\ + \beta\, \mathbb{E}[v_{N-2}^T P_{N-1} v_{N-2}]. \end{aligned} \tag{7.36}$$

By substituting (7.36) in (7.33) and observing that $P_k = G_k + \beta A^T P_{k+1} A$, we obtain

$$\begin{aligned} V_{N-2}(I_{N-2}^{TCP}) = \mathbb{E}[x_{N-2}^T K_{N-2} x_{N-2} | I_{N-2}^{TCP}] \\ + \mathbb{E}[e_{N-2}^T P_{N-2} e_{N-2} | I_{N-2}^{TCP}] \\ + \mathbb{E}[v_{N-2}^T (K_{N-1} + \beta P_{N-1}) v_{N-2}] \\ + \mathbb{E}[v_{N-1}^T K_N v_{N-1}] + 2\lambda. \end{aligned} \tag{7.37}$$

Proceeding similarly at stages $N-3,\ N-4, \ldots,\ 0$, we obtain the overall cost as in (7.30). □

Remark 7.1. Note that due to the information structure of a TCP-like protocol the controller and the estimator are structurally different from the controller and the estimator under a UDP-like protocol.

Theorem 7.2. *Consider the system (7.3) with control and scheduling policy π_{TCP} defined as*

$$(u_k, \sigma_k) = \begin{cases} (L_k \hat{x}_k, 1), & \text{if } \hat{x}_k^T \Gamma_k \hat{x}_k > \lambda \\ (\emptyset, 0), & \text{otherwise} \end{cases} \tag{7.38}$$

$$\Gamma_k = \bar{\alpha} A^T K_{k+1} B (R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A,$$

where

$$\hat{x}_k = \begin{cases} A\hat{x}_{k-1} + \alpha_{k-1} B\sigma_{k-1} u_{k-1} & \text{if } \beta_k = 0 \\ x_k & \text{if } \beta_k = 1. \end{cases} \tag{7.39}$$

Then the policy π_{TCP} satisfies

$$J_0^{\pi_{TCP}}(I_0^{TCP}) \leq J_0^{\pi_{all}}(I_0^{TCP}) + N\lambda. \tag{7.40}$$

Proof. The proof follows similar steps as the proof of Theorem 7.1 and is removed for the sake of brevity. □

Ideal Networks

Another variant of the considered procedure happens when both actuator and sensor networks are ideal, i.e., $\beta_k = \alpha_k = 0,\ k \in \mathbb{S}$, then the available information at the controller is characterized by (7.10) and we have $y_k = x_k\ k \in \mathbb{S}$. Furthermore, the actuation signal (7.2) becomes $\hat{u}_k = \sigma_k u_k,\ k \in \mathbb{S}$. If transmission is triggered at any time instant i.e. $\sigma_k = 1,\ k \in \mathbb{S}$, then the optimal control policy is simply the solution of the well-known LQG problem $u_k = L_k x_k$ with

$$\begin{aligned} L_k &= -(R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A \\ K_k &= Q + A^T K_{k+1} A - P_{k+1} \\ P_k &= A^T K_{k+1} B (R + B^T K_{k+1} B)^{-1} B^T K_{k+1} A \\ K_N &= Q_N, \end{aligned} \tag{7.41}$$

which leads to the performance

$$V_0^{\pi_{all}}(I_0^I) = \mathbb{E}[x_0^T K_0 x_0 | I_0^I] + \sum_{k=1}^{N-1} \mathrm{Tr}(K_k \Phi_v) + N\lambda. \tag{7.42}$$

Theorem 7.3. Consider the system (7.1) with the ETC policy π_{LL} defined as

$$(u_k, \sigma_k) = \begin{cases} (L_k x_k, 1), & \text{if } \ x_k^T P_k x_k > \lambda \\ (\emptyset, 0), & \text{otherwise} \end{cases} \tag{7.43}$$

then the closed-loop system acquires the performance

$$J_0^{\pi}(I_0^I) \leq J_0^{\pi_{all}}(I_0^I) + N\lambda. \tag{7.44}$$

Proof. The proof follows similar steps as the proof of Theorem 7.1 and is removed for the sake of brevity. □

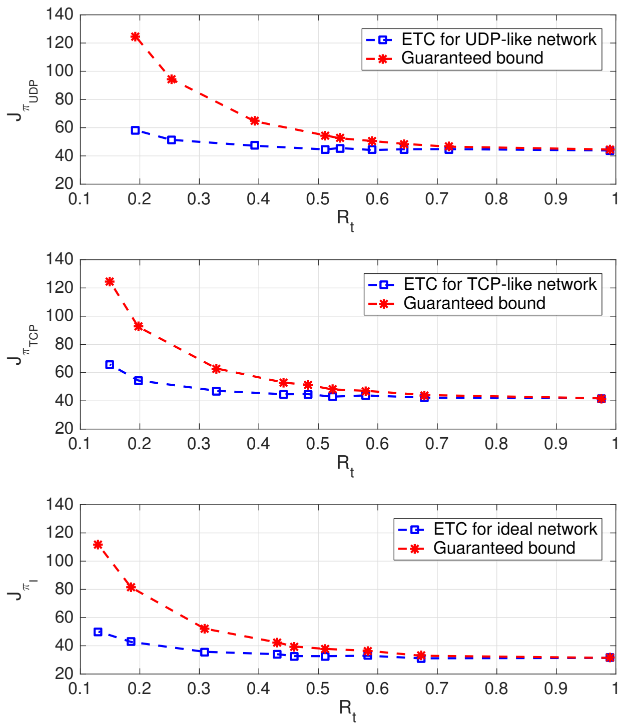


Figure 7.2. Performance of the proposed ETC policies under unreliable network links. The vertical axis represents the performance in terms of (7.6) for different information structure, on top the network with a UDP-like protocol, in middle the network with a TCP-like protocol and at the bottom an ideal network. The horizontal axis represents the average triggering rate as defined in (7.13).

7.4 Simulation results

We consider a discretized model of a double integrator controlled over two networks as depicted in Figure 7.1. The discretized system with the sampling time of 0.1 (sec) and the cost parameters are given by

$$A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix},$$

$$\Phi_v = 0.01, \quad N = 100, \quad Q_N = Q = I, \quad R = 1.$$

In this example, we consider the drop-out probabilities of $\beta = 0.6$ and $\alpha = 0.3$ for the sensor and the actuator networks, respectively. Figure 7.2 shows the simulation results for the three different policies for $\lambda \in [0,\ 1]$ based on Monte Carlo simulation of 600 realizations. As can be seen all the proposed policy operated within the guaranteed analytical performance bounds.

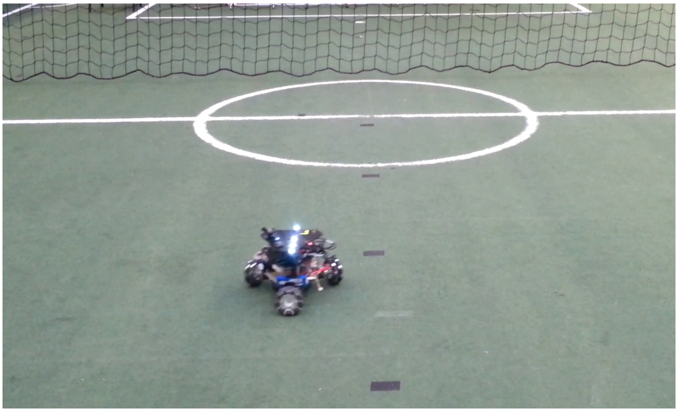


Figure 7.3. The omni-directional robot used for the experiments.

7.5 Experimental validation for remote control of a ground robot

In this section, we use the developed algorithm for model-based tracking and remote regulation of a ground robot towards a predefined trajectory. The experimental set-up consists of an omni-directional robot, see Figure 7.3, a top camera above a soccer field, where the robot is moving and a controller on a host PC. Images of the robot on the soccer field are sent from the top camera to the controller, these images are processed to obtain the position and orientation of the robot with respect to the world frame. Then based on the calculated position control actions are computed as will be discussed in the sequel.

Figure 7.4 shows a schematic representation of the control structure. The plant represents the omni-directional robot, whose dynamics can be described by a linear model The robot model is given by the following discrete-time linear time-invariant system

$$\begin{aligned}
\underbrace{\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \psi_{k+1} \end{bmatrix}}_{X_{k+1}} &= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_k \\ y_k \\ \psi_k \end{bmatrix}}_{X_k} + T_s \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \hat{u}_{x,k} \\ \hat{u}_{y,k} \\ \hat{u}_{\psi,k} \end{bmatrix}}_{\hat{u}_k} + v_k \\
y_k &= \begin{cases} X_k, & \beta_k = 1, \\ \emptyset, & \text{otherwise}, \end{cases}
\end{aligned} \tag{7.45}$$

where T_s is the sampling time, which is set to 0.1 second. The control input $\hat{u}_k$ consists of a feedforward term $\hat{u}_k^r$ and a feedback term $\hat{u}_k^z$. The feedforward term is computed from the desired model-based trajectory $X_k^r,\ k \in \mathbb{S}$, and obtained

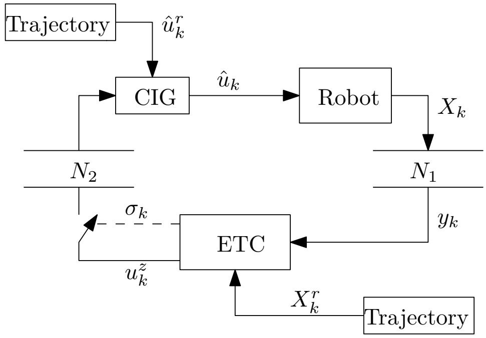


Figure 7.4. Block diagram of a controller which combines state feedback and feedforward from a trajectory generator. The feedforward command u_k^r along with the desired state X_k^r are calculated a priori and stored on both the robot and the ETC unit. The feedback controller uses the received state y_k and the reference trajectory X_k^r to compute a corrective input u_k^z.

through

$$X_{k+1}^r = AX_k^r + B\hat{u}_k^r. \tag{7.46}$$

The feedback term, on the other hand, regulates the robot towards the desired trajectory in the presence of disturbances. Since the system is linear we can use the proposed ETC policies to design the feedback control input based on the error between the position of the robot X_k and the reference, X_k^r which has the dynamics

$$z_{k+1} = Az_k + B\hat{u}_k^z + v_k,$$

where $z_k = X_k - X_k^r$. We assume that both the remote controller and the robot have access to the feedforward control inputs and the controller only sends the feedback input to the robot. Therefore, the control input is obtained by

$$\hat{u}_k = \hat{u}_k^z + \hat{u}_k^r, \tag{7.47}$$

where

$$\hat{u}_k^z = \sigma_k \alpha_k u_k^z. \tag{7.48}$$

Therefore, if no control input is received by the robot either because of packet dropout $\alpha_k = 0$ or because the triggering condition is not active, i.e., $\sigma_k = 0$, then the feedforward term $\hat{u}_k^r$ is still applied to the actuators, i.e., only the feedback term $\hat{u}_k^z$ of the input $\hat{u}_k$ is set to zero.

Experimental results

In the experiments, we consider the performance index (7.6) with

$$Q = I_3, \quad Q_N = I_3, \quad R = I_3, \quad N = 250.$$

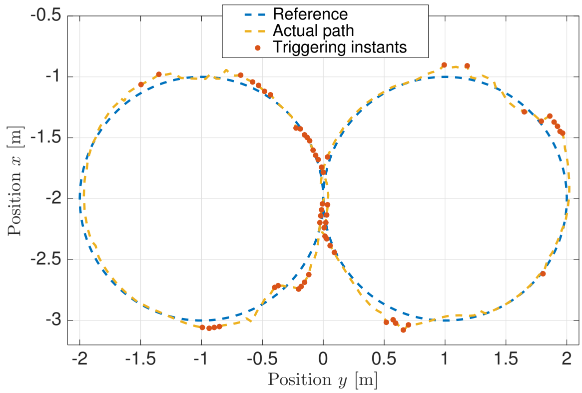


Figure 7.5. The tracking in the x, y-plane for the proposed event-triggered mechanism (7.22) with threshold $\lambda = 0.01$, only the tracking for the first perambulation is shown. The instants at which a triggering occurs are depicted with red dots.

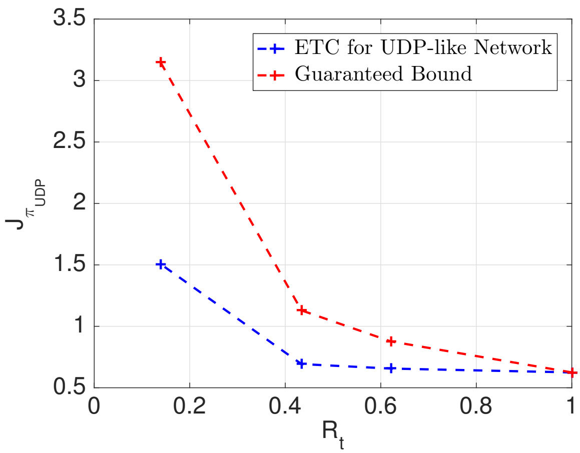


Figure 7.6. Performance of the proposed ETC in the considered experimental setup where $J = \mathbb{E}[z_N^T Q_N z_N + \sum_{k=0}^{N-1} z_k^T Q z_k + \hat{u}_k^{z^T} R \hat{u}_k^z]$.

The actual percentage of packets that was lost in the actuator network is approximately 1%. This percentage is very low and it would not show the usefulness of the event-triggered policies. Therefore, randomly induced packet drops were introduced to acquire a drop-out probability of $\alpha = 0.1$. In the sensor network the packet drop-outs were mainly due to missing the computation deadline. This leads to a dropout probability corresponding to $\beta = 0.25$. For this experiment we only consider the UDP-like protocol thus no acknowledgment is available in the actuator network.

Since the computation of a (new) feedback control value can not be done instantaneously at sampling times, we introduce a unit delay for applying controller actions and, therefore, we update the state estimation $\hat{z}_k$ by prediction i.e.,

$$u_k = L_k(A\hat{z}_{k-1} + \bar{\alpha}Bu_{k-1}).$$

The reference trajectory consists of two tangent circles with radius of 1 meter as depicted by blue dashes in Figure 7.5. Moreover, the real trajectory and transmission instants of the remote robot are also depicted in Figure 7.5.

Figure 7.6 shows the experimental results for four different values of $\lambda \in \{0,\ 0.001,\ 0.002,\ 0.01\}$. Each experiment was carried out for 5 rounds and the obtained performance is averaged over these experiments. Interestingly, the performance of the proposed ETC policy, J^{π}, is always below the guaranteed bounds, $J^{\pi_{all}}$, which validates the proposed scheme in the experimental set-up.

7.6 Conclusions

A suboptimal ETC policy with guaranteed performance under unreliable actuator and sensor links was provided in this chapter. The proposed policy belongs to the class of threshold-based policies, whose parameters are influenced by the underlying characteristics of the packet dropout behaviour in the communication network and the cost of communication in the considered quadratic performance index. An experimental setup for validation of the proposed algorithm has been developed. In the experiment a remote controller is used to steer an omni-directional ground robot, along a predefined trajectory. The conducted experiments validated the theoretical results and the applicability of the proposed ETC policy.

Chapter 8

Conclusions and Recommendations

The emergence of cost-effective and reliable sensors and actuators with wireless/wired networking capabilities has enabled the manipulation of the physical world through such "cyber" devices at an unprecedented scale. This gave birth to so-called cyber-physical systems. As it is the case with the advent of any new concept or technology, the birth of cyber-physical systems asks for a paradigm shift in several research fields. This was the case for control and communication sciences, which resulted in a new research field called networked control systems (NCSs).

One of the main challenges in NCSs is the design of tools for the efficient management of energy, communication, and computation resources. Periodic time-triggered sampling and execution has been the major communication and computation resource management paradigm for decades. However, periodic strategies lack the flexibility one needs to manage these resources efficiently. As an answer to this need, event-triggered control (ETC) has been proposed and has attracted many researchers in the last decade. The main idea behind ETC is to include the information of the state and/or the output of the system in the determination of data-transmission times (communication resource management) or execution times (computation resource management) in a control loop. Therefore, in a networked control setting, one can consider the ETC unit as an agent trying to create a balance between the performance of the system and the use of the feedback resources. ETC solutions can be classified in two design approaches. Some use Lyapunov-based methods typically for nonlinear systems (later also using hybrid systems) and some optimization-based approaches typically for linear systems with stochastic disturbances. The latter approach often considers co-design of triggering and control policies by using a (typically quadratic) cost function, penalizing both communication resources and control actions. Optimal ETC design addresses the problem of characterizing the control and scheduling policies that minimize the considered performance criteria. However, computing optimal ETC policies is typically intractable. For example, many approaches cast the problem in a dynamic programming formulation that is limited by the curse of dimensionality. Suboptimal ETC design, as considered in this thesis, on the other hand, focuses on finding simple and easy to implement control and scheduling policies that have guaranteed performance and therefore try to overcome the bottlenecks that arise with the optimal ETC design.

8.1 Conclusions

In this dissertation, we considered three questions as the main focal points of our research.

(i) *How to design output-based ETC policies with performance guarantees?*

(ii) *How to develop policies that not only have guaranteed performance but also outperform the periodic time-triggered policies in terms of quadratic performance for the same average transmission rate?*

(iii) *How to experimentally validate the developed results and deal with practical features such as packet dropouts in real scenarios?*

In what follows we give an overview of our approach to answer these questions.

(i) How to design output-based ETC policies with performance guarantees?

In Chapter 2, we proposed an optimization-based output-feedback ETC solution for linear discrete-time systems with guaranteed performance expressed in terms of the optimal periodic (all-time) control performance, while reducing the communication load. The performance was measured by an average quadratic cost. Several connections with previous works in the literature have been established, and in particular, with the absolute, relative and mixed threshold policies [33,44,79]. The usefulness of the results was illustrated through a numerical example showing that a significant (up to 72%) reduction in network usage can be achieved by only sacrificing 10% of performance compared to the optimal all-time transmission policy. Furthermore, we showed that our proposed ETC can lead to significant improvements in the performance at the same average transmission rate when compared to optimal time-triggered *periodic* controllers.

In Chapter 3, we built upon the presented results in Chapter 2 and considered multiple channels by including a sensor-to-actuator network. Our objective was to design an event-triggering mechanism for each channel while guaranteeing a performance bound as in Chapter 2. We proposed a simple, easy-to-implement ETC policy for linear discrete-time systems in this more general multi-channel configuration. The considered setup consists of an NCS in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. Our proposed ETC method was obtained via an optimization-based scheme in which the performance is measured by a quadratic cost. We showed that the resulting policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions. The usefulness of the results was illustrated through two numerical examples.

(ii) How to develop policies that not only have guaranteed performance but also outperform the periodic time-triggered policies in terms of quadratic performance for the same average transmission rate?

In Chapter 4, we proposed two consistency notions for an ETC policy and provided a policy for continuous-time linear systems that satisfies them. The proposed policy was derived based on the optimal periodic control average cost, viewed as a function of the sampling period, and was illustrated via simulations. We also provided an example of a linear quadratic control problem for which a traditional ETC policy, where transmissions occur if the Euclidean norm of the error between the system's state and a state estimate exceeds a threshold, is not consistent in the sense that it does not achieve a better closed-loop performance than the traditional periodic control for the same average transmission rate.

In Chapter 5, we showed that, in linear systems, the trade-off curve between the average quadratic cost and the transmission rate for the optimal periodic control is convex. Building upon this fact, we proposed a periodic event-triggered policy that has both consistency properties. The resulting policy is an absolute threshold policy relying on a weighted norm. The weights are tuned with respect to the parameters of the considered quadratic performance. The proposed policy can be classified as a suboptimal ETC policy which outperforms the periodic time-triggered control both in network resource utilization and average quadratic cost.

(iii) How to experimentally validate the developed results and deal with practical features such as packet dropouts in real scenarios?

In Chapter 6, we validated the ETC for linear discrete-time systems derived in Chapter 3 on a motion system consisting of a ground robot controlled over a communication network to realize remote regulation. The experiment reveals that the proposed ETC policy significantly reduces the communication in the sensors' and the actuators' networks by 80% and 90%, respectively, while guaranteeing performance bounds on the cost. The proposed policy can not deal with packet dropouts and, therefore, these are neglected in the validation.

In Chapter 7, we address the design and validation of a suboptimal ETC policy with guaranteed performance under unreliable actuators' and sensors' links. The proposed policy belongs to the class of absolute threshold policies relying on a weighted norm. The weights are determined by the underlying characteristics of the packet dropouts in both communication channels and the considered quadratic performance index. An experimental setup for validation of the proposed algorithm has been developed. The setup consists of a ground omni-directional robot controlled over unreliable sensors' and actuators' links to follow a predefined trajectory. As it was shown in the experiments, a reduction of up-to 88% in the triggering rate at the actuator network can be achieved while both sensors' and actuators' links are prone to packet dropouts with probabilities of 0.1 and 0.25, respectively.

8.2 Recommendations for Future Research

There are still many directions to explore in the appealing and challenging area of suboptimal ETC strategies. As a result of the work done in this dissertation, following directions could be considered.

Multi-agent suboptimal ETC strategies: In this dissertation, we mainly focused on the design of suboptimal ETC policies for single-agent control systems. However, many applications consist of multi-agent control problem in which each agent has different control objectives. Since achieving optimality in the presence of several decision makers (agents) is not tractable in most problems, these suboptimal ETC design approaches can be promising in this context. In fact, due to the more relaxed problem formulations that focus on achieving a *good* performance instead of an optimal one, suboptimal ETC strategies seem to provide a promising avenue to address performance-based scheduling and control for a multi-agent control problem. Another interesting open question with respect to the results presented in Part II of this dissertation is, how to extend the definition of consistency for ETC in a multi-agent setting. We believe that this definition will help characterizing suboptimal ETC that can outperform the time-triggered control policies for resource management.

Considering nonlinear systems: Although as mentioned earlier many works in the literature deal with the design of ETC strategies for the general class of nonlinear systems, the main focus of these research efforts has been on stability-based performance criteria like Lyapunov stability or $\mathcal{L}_p$-gain stability. Moreover, these works often use emulation-based design which introduces some conservativeness in the design of ETC. However, the optimization-based approach, which has clear advantages due to their focus on performance and also the inherent potential of addressing the co-design problem, has not attracted much attention yet. Therefore, an interesting open question in this context is how to design (sub)optimal ETC strategies for nonlinear (stochastic) systems. The challenges include, but are not limited to, specifying the performance index, the class of stochastic processes that are relevant to consider, and the identification of application areas in which these policies can be beneficial.

Continuing the experimental exploration: In this dissertation, we took a step in validating ETC policies in remote control of a single ground robot. Furthermore, we designed and implemented ETC policies for networks with unreliable communication links. However, overall it is fair to say that the experimental exploration of ETC solutions is still in its infancy (see, e.g., [21, 92–96]). Therefore, in order to steer the research in ETC community towards more applicable solutions, we encourage the community to start validating existing solutions in laboratory settings. Hopefully, a collection of these experimental explorations would help bringing the theory and application closer and, reveal important questions that need to be addressed thereby fueling new theoretical research in the area.

8.3 Final Thoughts

With this dissertation, we have provided new contributions for the analysis, design, and experimental validation of resource-aware control solutions classified under ETC in an optimization-based setting. We believe that these results can be important cornerstones in bringing ETC to more real-life applications. In fact, we showed the potential that ETC can have in two robotic applications. In the introduction of this dissertation, we addressed some exciting areas of technology that can benefit from the use of ETC mechanisms. As the concept of IoT has started to influence many aspects of human life and, therefore, effective resource management concerning communication, computation, and energy resources has become indispensable. As such, the accomplishments carried out in this dissertation can contribute to this goal.

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Societal Summary

We are living in an information-rich era, where almost every aspect of our lives is influenced by ubiquitous information networks. Physical objects -from cars to watches- are equipped with sensors and actuators that are linked through wired and wireless networks creating a medium of connected devices known as the Internet of Things (IoT). This situation is unparalleled in human history and, as a consequence, the study of networks and their effects have become a scientific, and technological imperative for the 21st century. This is also the case for control science. There the question is how to control these interacting networked objects in order to obtain a desired performance (such as efficient fuel consumption for a car) in a dynamic environment.

In fact, the introduction of communication networks in control systems gave rise to the growing field of networked control systems (NCSs). In a NCS, the efficient utilization of communication and energy resources is a key challenge. This has lead to the introduction of integrated control and resource management solutions such as event-triggered control (ETC), which uses feedback control not only to realize proper performance of the overall systems but also to efficiently manage the communication resources (in terms of bandwidth). In particular, in an ETC scheme, the communication resources are managed based on the current status of the system such that these resources are only utilized when necessary from the system's performance view point. This leads to a trade-off between the use of the network and the performance of the system. In this dissertation, we investigate this trade-off and propose novel algorithms that can be tuned to achieve a desired trade-off in a user-friendly manner. Furthermore, through experiments with ground robots, we demonstrate the application potential of the conducted research, which can be beneficial in many other situations including vehicle platooning, cooperative robotics and drones, smart grids and buildings, and so on.

Summary

Event-triggered Control for Linear Systems with Performance and Rate Guarantees: An Approximate Dynamic Programming Approach

Efficient utilization of communication, computation and energy resources is one of the key challenges in many emerging networked control applications. Periodic sampling and control is the most common approach in control systems technology but lacks the flexibility to utilize these resources in an efficient way. Recent research proposes to depart from the periodic control paradigm in favor of event-triggered control (ETC). The fundamental idea behind ETC is that transmissions should be triggered by events inferred from the state or the output of the plant. This leads potentially to an improvement of the trade-off between the average transmission rate and the control performance when compared to periodic control, since in ETC the utilization of resources can be scheduled in a smarter way. Desirably, the communication protocols corresponding to ETC should still be insightful and simple to implement and guarantee important performance properties for the control system. As the system theory on ETC matures, there is also a need to experimentally validate and test these methods in applications of interest.

In this dissertation, we propose novel event-triggered control policies with guarantees on the closed-loop performance and on the transmission rate. Performance is mostly assessed via a quadratic cost penalizing deviations from the desired state and control values. Our approach is based on techniques such as approximate dynamic programming leading to sub-optimal solutions, motivated by the fact that optimal solutions for ETC problems are typically computationally intractable. Following this approach, we are still able to characterize the control performance and the average transmission rate. In particular, we can guarantee that the proposed ETC policies outperform periodic control for the same transmission rate. In this context, we considered three questions as the main focal points of the conducted research:

(i) *How to design output-based ETC policies with performance guarantees?*

In the first part of the dissertation, we propose an output-based ETC solution for linear discrete-time systems with a performance guarantee relative to periodic time-triggered control, while reducing the communication load. The performance is expressed as an average quadratic cost and the plant is disturbed by Gaussian process and measurement noises. This line of research was further extended by considering a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The new policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions.

(ii) *How to develop policies that not only have guaranteed performance but are also guaranteed to outperform the periodic time-triggered policies in terms of quadratic performance for the same average transmission rate?*

To answer this question, in the second part of this dissertation we propose ETC policies that are *consistent* in the sense that they (a) achieve a better closed-loop performance (quadratic cost) than the traditional periodic control for the same average transmission rate and (b) do not generate transmissions in the absence of disturbances. Moreover, we extended this research line towards consistent periodic ETC policies in which we propose a simple threshold event-triggered policy that is consistent.

(iii) *How to experimentally validate the developed results and deal with practical features such as packet dropouts in real scenarios?*

In the last part of this dissertation, the proposed ETC schemes are experimentally validated in the context of control of ground robots. This includes the remote sensing and the control of a wireless robot moving along a one-degree of freedom and the event-triggered feedback control of an omni-directional robot over unreliable wireless networks.

Summarizing, we provided new contributions for the analysis, design, and experimental validation of resource-aware control solutions classified under ETC in an optimization-based setting. We believe that these results can be important cornerstones in bringing ETC to many real-life applications.

Curriculum Vitae

Behnam Asadi Khashooei was born on June 26, 1987, in Tehran, Iran. He received his Bachelor of Science (Honors) and Master of Science (cum laude) degrees from the department of Electrical and Computer Engineering (ECE) at Isfahan University of Technology (IUT), Isfahan, Iran, in 2009 and 2012, respectively. His master's thesis focused on hybrid observer design for a class of nonlinear hybrid systems with an application in frequency estimation of saturated sinusoidal signals.

Since April 2013, he started his Ph.D. project within Control Systems Technology group of the department of Mechanical Engineering at the TU/e, under the guidance of Maurice Heemels and Duarte Antunes. His research project was part of the VICI project "wireless control systems: A new frontier in automation" funded by NWO Applied and Engineering Sciences, and was focused on the analysis and design of event-triggered controllers with performance guarantees for networked control systems. The results of his research are printed in this dissertation.

We are living in an information-rich era, where almost every aspect of our lives is influenced by ubiquitous information networks. Physical objects -from cars to watches- are equipped with sensors and actuators that are linked through wired and wireless networks creating a medium of connected devices known as the Internet of Things (IoT). This situation is unparalleled in human history and, as a consequence, the study of networks and their effects have become a scientific, and technological imperative for the 21st century. This is also the case for control science. There the question is how to control these interacting networked objects in order to obtain a desired performance (such as efficient fuel consumption for a car) in a dynamic environment.

In this dissertation, we investigate the trade-off between the use of the network and the performance of a system controlled over a communication network. We propose novel algorithms that can be tuned to achieve a desired trade-off in a user-friendly manner. Furthermore, through experiments with ground robots, we demonstrate the application potential of the conducted research, which can be beneficial in many other situations including vehicle platooning, cooperative robotics and drones, smart grids and buildings, and so on.